Central force fields generated by m theory

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Abstract

The introduction of m theory was a replacement of Einstein's erroneous field equation. The new theory was applied in the macroscopic realm to problems of gravitation by using Lagrange theory. In this paper, we extend the underlying formalism to the full range of Cartan geometry, obtaining all internal quantities like spin connections for a given potenital. The results are valid for both electromagnetic and gravitational central structures. We obtain a new central force that stems from the geometric structure of spacetime itself. In addition, a rotational field appears, although there are no rotational parts in the potential. The approach of m theory, which is based on the line element of general relativistic spacetime, can be generalized without essential changes of the results.

Keywords: Unified field theory; m theory; central symmetry; gravitation; electromagnetism.

1 Introduction

ECE theory is the physical interpretation of Cartan geometry [1–5] and has been developed since 2003. It replaces Einstein's theory of general relativity by introducing torsion into theoretical physics. Torsion was inferred by Cartan as a completion of Riemann geometry that only contains curvature. Torsion cannot be neglected, because curvature is always connected with torsion. Setting torsion to zero leads to contradictions. Therefore, Einstein's field equation is no more tenable and can only be considered as an approximation in weak fields, where it transits into ECE theory under certain preconditions [7].

What retains value, however, is the metric of space-time describing curvilinear coordinates as found in Einstein's general theory of relativity and also in ECE theory. The m theory [5] was derived from this metric. It describes the distortion of the coordinates in a centrosymmetric geometry, as it is often used in cosmology, and has also been applied in the quantum world. By m theory, the unification of the quantum theory with general relativity was achieved [6].

In [5], the metric of m theory has been combined with Lagrangian mechanics. Valuable results in the field of relativistic mechanics have been obtained from

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this approach. In particular, the deflection of light by heavy celestial objects could be described correctly and parameter-free. In this paper, we extend the possible uses of m theory. In order to cover the complete range of physics described by Cartan geometry, we fully integrate m theory into this geometry. To do this, the metric must be expressed by the Cartan tetrad. Then, the full range of physics covered by the Cartan geometry can be calculated, from the potentials to the Cristoffel symbols and spin connections to the force fields of mechanics and electrodynamics.

It has already been shown in [5] that the diagonal metric of m theory can be converted into a diagonal tetrad structure. To this tetrad we then apply the complete calculation mechanism of Cartan geometry as described in [5] and [8]. This gives us interesting insights, when the m function goes to zero at an event horizon or at the center of the coordinate system. The function $\mathrm{m}(r)$ describes a radial space density or aether density. A significant decrease in this density leads to particular effects.

2 Connection of the general relativistic line element with Cartan geometry

Within this paper, we will use the metric of a non-constant, centrally symmetric spacetime that is different from Minkowski space. We will base our development on a metric that is common in Einstein theory, but we will develop our method within ECE2 theory, i.e., Cartan geometry. Nonetheless, this a development of "true general relativity", even in the sense of standard physics.

According to Section 2.1.3 of [5], the squared line element in a space with curvature and torsion is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{1}$$

where $g_{\mu\nu}$ is the symmetric metric and dx^μ is the differential of the spacetime coordinate x^μ. In a Minkowski space for a spherically symmetric spacetime, this takes the form

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin(\theta)^2 d\phi^2 \tag{2}$$

with a time coordinate $x_0 = ct$, radius coordinate r, polar angle θ and azimuthal angle ϕ. In a general spherically symmetric spacetime with torsion and curvature, the line element has to be generalized as described in Chapter 7 of [9]:

$$ds^2 = c^2 \mathrm{m}(r,t) dt^2 - \mathrm{n}(r,t) dr^2 - r^2 d\theta^2 - r^2 \sin(\theta)^2 d\phi^2. \tag{3}$$

$\mathrm{m}(r,t)$ and $\mathrm{n}(r,t)$ are general functions describing the distortion of spacetime by a central point mass at $r = 0$. Only the radial and time coordinates are affected. The angular parts remain unchanged because of the rotational symmetry. It was shown in [9] that the line element can be simplified further by the replacement

$$\mathrm{n}(r,t) = \frac{1}{\mathrm{m}(r,t)}, \tag{4}$$

and that the time dependence of $\mathrm{m}(r,t)$ can be rolled over to the time coordinate. Therefore, the simplified line element reads

$$ds^2 = c^2 \mathrm{m}(r) dt^2 - \frac{dr^2}{\mathrm{m}(r)} - r^2 d\theta^2 - r^2 \sin(\theta)^2 d\phi^2. \tag{5}$$

This form is used in Einstein's field equation, in which the Ricci tensor is zero and an expression is derived for for $\mathrm{m}(r)$ in the vacuum, leading to the Schwarzschild metric. In ECE2 theory, we use this form for simplicity, but we can freely define the function $\mathrm{m}(r)$. Comparing Eq. (1) with Eq. (5), it follows that the metric is diagonal and the metric coefficients are

$$g_{00} = \mathrm{m}(r), \quad g_{11} = -\frac{1}{\mathrm{m}(r)}, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin(\theta)^2. \tag{6}$$

This metric-based theory, which we have called *m theory* [5], can also be developed from Cartan geometry itself. In Cartan geometry, the basis element is the tetrad, and the metric follows from the tetrad [5] by

$$g_{\mu\nu} = n \; q^a{}_\mu q^b{}_\nu \, \eta_{ab}, \tag{7}$$

where $q^a{}_\mu$ are the tetrad elements, η_{ab} is the Minkowski metric of tangent space, and $n = 4$ is the dimension of the base manifold. The metric does not generally allow the tetrad to be determined uniquely. In this case, however, the metric is diagonal. We can assume that the tetrad matrix is diagonal also, because we do not consider specific polarization effects of Cartan geometry. Therefore, Eq. (7) reduces to the diagonal elements in both the base manifold and tangent space, and we obtain

$$q^{(0)}{}_0 = \frac{1}{2}\sqrt{\mathrm{m}(r)}, \quad q^{(1)}{}_1 = \frac{1}{2\sqrt{\mathrm{m}(r)}}, \quad q^{(2)}{}_2 = \frac{r}{2}, \quad q^{(3)}{}_3 = \frac{r\sin(\theta)}{2}. \tag{8}$$

This is the connection of m theory to Cartan geometry.

The position vector in m space is

$$\mathbf{r} = \frac{r}{\mathrm{m}(r)^{1/2}} \mathbf{e}_r, \tag{9}$$

and the velocity in m space with two spatial dimensions is

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{1}{\mathrm{m}(r)^{1/2}} \left(\dot{r}\mathbf{e}_r + r\dot{\phi}\mathbf{e}_\phi \right). \tag{10}$$

Sometimes, we use the variable name

$$\mathbf{r}_1 = r_1 \mathbf{e}_r = \frac{r}{\mathrm{m}(r)^{1/2}} \mathbf{e}_r, \tag{11}$$

so that the velocity becomes

$$\mathbf{v}_1 = \dot{\mathbf{r}}_1 = \dot{r}_1 \mathbf{e}_r + r_1 \dot{\phi} \mathbf{e}_\phi. \tag{12}$$

A new time variable can be defined by

$$t_1 = \mathrm{m}(r)^{1/2} t. \tag{13}$$

$\mathbf{r}_1$ and t_1 are the characteristic variables of m space.

From the line element (5) of m space, it follows that

$$ds^2 = c^2 \mathrm{m}(r) dt^2 - \left(\frac{d\mathbf{r}_1}{dt} \right)^2 dt^2 = c^2 dt_1^2 - \mathbf{v}_1^2 dt^2. \tag{14}$$

In plane polar coordinates related to the observer space, this becomes

$$ds^2 = c^2 \left(\mathrm{m}(r) - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{\mathrm{m}(r)c^2} \right) dt^2 \tag{15}$$
$$= \frac{c^2 dt^2}{\gamma^2}.$$

Thus, the general relativistic γ factor of m space is defined by

$$\gamma = \left(\mathrm{m}(r) - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{\mathrm{m}(r)c^2} \right)^{-1/2}. \tag{16}$$

The linear momentum of m space is

$$\mathbf{p}_1 = \gamma m \mathbf{v}_1 = \gamma m \frac{\mathbf{v}}{\mathrm{m}(r)^{1/2}}. \tag{17}$$

3 Computational basis and examples

In [8], it has been shown how all Christoffel symbols, spin connetions, curvature and torsion tensors are derived frome the Cartan tetrad. Here, we only repeat the variable names and their denominations:

${\Gamma^\rho}_{\mu\nu}$:	Christoffel connection
${\omega^a}_{\mu b}$:	spin connection
${R^\lambda}_{\rho\mu\nu}$:	Riemann (curvature) tensor
${T^\lambda}_{\mu\nu}$:	torsion tensor
${R^a}_{b\mu\nu}$:	curvature form
${T^a}_{\mu\nu}$:	torsion form
$\mathbf{E}^a$:	electric field
$\mathbf{B}^a$:	magnetic field
${\Lambda^\lambda}_{\mu\nu}$:	dual Christoffel connection
${\omega_{(\Lambda)}}^a{}_{\mu b}$:	dual spin connection

According to Eq. (8), the tetrad matrix is

$$({q^a}_\mu) = \begin{bmatrix} \frac{1}{2}\sqrt{\mathrm{m}(r)} & 0 & 0 & 0 \\ 0 & \frac{1}{2\sqrt{\mathrm{m}(r)}} & 0 & 0 \\ 0 & 0 & \frac{r}{2} & 0 \\ 0 & 0 & 0 & \frac{r\sin(\theta)}{2} \end{bmatrix}, \tag{18}$$

and from this follows the metric:

$$(g_{\mu\nu}) = \begin{bmatrix} \mathrm{m}(r) & 0 & 0 & 0 \\ 0 & -\frac{1}{\mathrm{m}(r)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin(\theta)^2 \end{bmatrix}. \tag{19}$$

The determinant of the metric is

$$\det(g_{\mu\nu}) = -\frac{1}{r^4 \sin(\theta)^2}. \tag{20}$$

Obviously, it is independent of the m function.

The execution of the computer algebra code developed in [8] gives all curvature/torsion parameters and connections. A larger number of them is zero, because the tetrad is diagonal. Some non-vanishing results are

$$\Gamma^0{}_{01} = -\Gamma^0{}_{10} = -\frac{\frac{d\mathrm{m}(r)}{dr}}{2\mathrm{m}(r)} \tag{21}$$

$$\Gamma^2{}_{12} = -\Gamma^2{}_{21} = \frac{1}{r} \tag{22}$$

$$\omega^{(0)}{}_{0(1)} = \omega^{(1)}{}_{0(0)} = -\frac{\frac{d\mathrm{m}(r)}{dr}}{2} \tag{23}$$

$$\omega^{(1)}{}_{2(2)} = -\omega^{(2)}{}_{2(1)} = \sqrt{\mathrm{m}(r)} \tag{24}$$

$$\Lambda^2{}_{03} = -\Lambda^3{}_{02} = \frac{1}{r} \tag{25}$$

$$\omega_{(\Lambda)}{}^{(2)}{}_{1(2)} = -\frac{1}{r} \tag{26}$$

$$R^0{}_{202} = -R^0{}_{220} = -\frac{r\frac{d\mathrm{m}(r)}{dr}}{2} \tag{27}$$

$$T^0{}_{01} = -T^0{}_{10} = -\frac{\frac{d\mathrm{m}(r)}{dr}}{\mathrm{m}(r)} \tag{28}$$

$$R^{(0)}{}_{202} = -R^{(0)}{}_{220} = -\frac{\sqrt{\mathrm{m}(r)}\frac{d\mathrm{m}(r)}{dr}}{2} \tag{29}$$

$$T^{(0)}{}_{01} = -T^{(0)}{}_{10} = -\frac{\frac{d\mathrm{m}(r)}{dr}}{2\sqrt{\mathrm{m}(r)}} \tag{30}$$

The resulting electric and magnetic force fields for the four polarization directions are

$$\mathbf{E}^{(0)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{d\mathrm{m}(r)}{dr}}{2\sqrt{\mathrm{m}(r)}} \\ 0 \\ 0 \end{bmatrix}, \tag{31}$$

$$\mathbf{E}^{(1)} = \mathbf{E}^{(2)} = \mathbf{E}^{(3)} = \mathbf{0}, \tag{32}$$

$$\mathbf{B}^{(0)} = \mathbf{B}^{(1)} = \mathbf{0}, \tag{33}$$

$$\mathbf{B}^{(2)} = \begin{bmatrix} 0 \\ 0 \\ -B_0 \end{bmatrix}, \qquad \mathbf{B}^{(3)} = \begin{bmatrix} -C_0 r \cos\theta \\ B_0 \sin\theta \\ 0 \end{bmatrix} \tag{34}$$

with constants A_0, B_0 and C_0. A_0 is the primordial vector potential $A^{(0)}$ of ECE theory, B_0 is a magnetic field constant and C_0 a constant with units Tesla/m.

Averaging the polarizations, we can write

$$\mathbf{E} = \mathbf{E}^{(0)} + \mathbf{E}^{(1)} + \mathbf{E}^{(2)} + \mathbf{E}^{(3)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{d\mathrm{m}(r)}{dr}}{\sqrt{\mathrm{m}(r)}} \\ 0 \\ 0 \end{bmatrix}, \tag{35}$$

$$\mathbf{B} = \mathbf{B}^{(0)} + \mathbf{B}^{(1)} + \mathbf{B}^{(2)} + \mathbf{B}^{(3)} = \begin{bmatrix} -C_0 r \cos\theta \\ B_0 \sin\theta \\ -B_0 \end{bmatrix}. \tag{36}$$

For gravitation, the **E** field corresponds to the gravitational field **g**, and the **B** field corresponds to the gravitomagnetic field $\mathbf{\Omega}$. The **E** field has a radial component only, while the **B** field has r, θ and ϕ components.

To demonstrate these fields graphically in a centrally symmetric geometry, we use the function $\mathrm{m}(r)$ that we have used earlier [5]:

$$\mathrm{m}(r) = 2 - \exp\left(\log(2)\exp(-\frac{r}{R})\right) \tag{37}$$

with a radial range R. This function, its derivative and the field component E_r are graphed in Fig. 1 with all constants set to unity. We have $\mathrm{m}(r) \to 0$ for $r \to 0$. However, the derivative of $\mathrm{m}(r)$ goes to a final limit. Consequently, the electric (or gravitational) field diverges when r approaches zero. This behaviour has already been identified in [5] as a "vacuum force" that appears by the radial variation of $\mathrm{m}(r)$. Such a force does not exist in classical physics and is a consequence of general relativity based on Cartan geometry. It is attractive and means that matter is pulled into the center as soon as it comes into the region where $\mathrm{m}(r)$ deviates from unity. In a sense, this may be interpreted as the ECE version of "black hoes" that are otherwise derived form Einsteinian general relativity in a mathematically wrong way.

Another interesting result is that a rotational magnetic (or gravitomagnetic) field exists. This is surprising, because the tetrad, which corresponds to the potential, has no rotational parts. The field is graphed in Fig. 2, showing the field vectors on two hemispheres. (The back side is omitted in order to not obscure visibility.) The figure shows a twist in each sphere. In principle, this represents a Torkado structure as was discussed in Example 8.15 of [5]. The Torkado has additionally a back-path at its central axis that does not appear in our example, but may also be present in a more customized geometry.

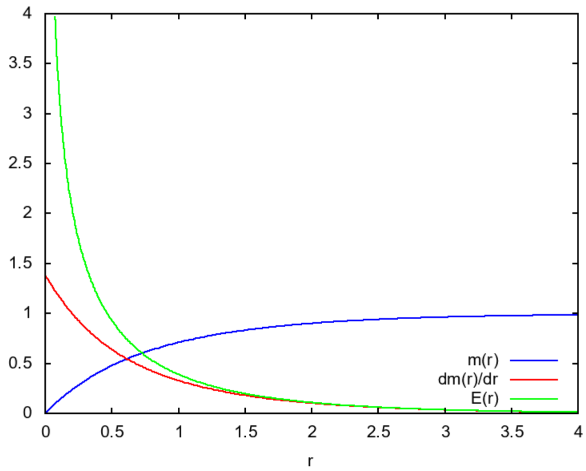

Figure 1: m(r), dm$(r)/dr$ and $E_r(r)$ for the model function m(r).

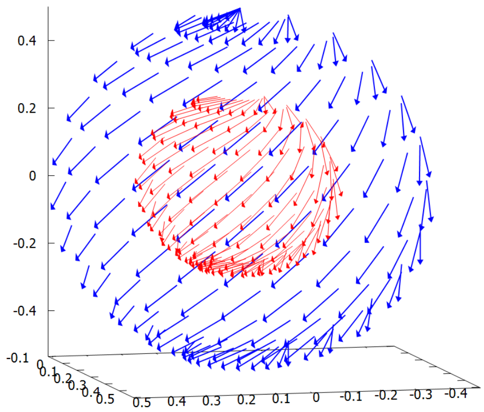

Figure 2: 3D representation of $\mathbf{B}(r)$ for the model function m(r).

4 Generalizations for spherical symmetry

So far, we have used the line element (5) of spherical symmetry. Therein, the approximation

$$\mathrm{n}(r) \approx \frac{1}{\mathrm{m}(r)} \tag{38}$$

was made for the function $\mathrm{n}(r)$ appearing in the more basic line element (3). Using both functions, the tetrad (18) reads

$$(q^a{}_\mu) = \begin{bmatrix} \frac{1}{2}\sqrt{\mathrm{m}(r)} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{\mathrm{n}(r)} & 0 & 0 \\ 0 & 0 & \frac{r}{2} & 0 \\ 0 & 0 & 0 & \frac{r\sin(\theta)}{2} \end{bmatrix}. \tag{39}$$

Evaluation by computer algebra gives the same results (31-34) for the force fields. Only the curvature and torsion parameters are affected. This means that Eq. (37) is not an approximation but an exact simplification of the line element. Another modification was to avoid the time dependence of the m function by rolling over it to the time coordinate. If we allow explicit time dependences in the form $\mathrm{m}(r,t)$ and $n(r,t)$, we obtain the original $\mathbf{E}$ field $\mathbf{E}^{(0)}$ plus an additional field $\mathbf{E}^{(1)}$:

$$\mathbf{E}^{(0)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{d\mathrm{m}(r,t)}{dr}}{\sqrt{\mathrm{m}(r,t)}} \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{E}^{(1)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{d\mathrm{n}(r,t)}{dt}}{\sqrt{\mathrm{n}(r,t)}} \\ 0 \\ 0 \end{bmatrix}. \tag{40}$$

The result for $\mathbf{E}^{(0)}$ is not changed, but $\mathbf{E}^{(1)}$ contains the time derivative of $\mathrm{n}(r)$ instead of the radial derivative of $\mathrm{m}(r)$. In the case of

$$\mathrm{n}(r,t) = \frac{1}{\mathrm{m}(r,t)}, \tag{41}$$

we obtain for the second $\mathbf{E}$ field polarization:

$$\mathbf{E}^{(1)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{d\mathrm{m}(r,t)}{dt}}{(\mathrm{m}(r,t))^{3/2}} \\ 0 \\ 0 \end{bmatrix}. \tag{42}$$

If only m depends on time but not n, the original result of Eqs. (31, 32) follows. We conclude that only a time dependence in the form $\mathrm{n}(r,t)$ leads to an additional $\mathbf{E}$ field.

We can further experiment by introducing non-diagonal terms in the tetrad. We have found that occupying the first row or first column by elements different from zero is a very critical choice. The equation system for solving the Christoffel symbols Γ is gives no solutions then in many cases. This means that couplings between time and space cannot be chosen arbitrarily and deserve high attention. Obviously, there are physical restrictions in realizing such couplings.

In summary, we have found a strong vacuum force in spherical symmetry. An additional rotational structure appears as a magnetic or gravitomagnetic field. The approximations of standard theory in the functions $\mathrm{m}(r)$ and $\mathrm{n}(r)$ are justified.

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