Can energy be extracted from a double pendulum?

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Abstract

According to statements by Milkovic and various inventors of rotoverter systems, it should be possible to extract energy from rotating systems with at least two revolving or oscillating units, which corresponds to a mechanism for extracting space energy. We base our work on the approach of an anonymous author who calculated the dynamics of a double pendulum according to classical mechanics. With a certain load characteristic, it gains energy. We have proven that the author did not use the underlying equations of motion correctly when applying an external load. If put into canonical, i.e., the intended way, there is no energy gain. We have investigated this for different types of load momenta. Such a system can only serve as a source of energy, if one adopts non-conventional physical mechanisms. For this, a "space-time resonance" was used according to the ECE theory. This then results in chaotic behavior, which, on average, leads to a significant gain in energy, with constant useful power being withdrawn from the system.

Keywords: double pendulum; Lagrangian mechanics; spacetime energy.

1 Introduction

In the context of research into alternative energy sources from space (vacuum energy, zero point energy, space energy), there are approaches that attempt to gain additional energy from mechanical systems. The first known system of this type was the Bessler wheel. More recently, the Milkovic [1] pendulum and the Würth [2] gearbox have become known. However, there is still no clear evidence of an energy gain with these devices. There are considerations of a more qualitative kind to make an energy gain plausible. These considerations come from engineers and inventors who mostly consider purely static configurations.

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Mechanical devices are dynamic systems because the internal forces and positions of the parts that make them up change over time. Therefore, a complete description of the functionality is only possible using dynamic considerations and models.

The dynamics of mechanical systems can be described very elegantly with methods of classical mechanics. If the models are structured simply enough, i.e. if it is a system of mass points, one uses the Lagrange theory, which goes back to Leonhard Euler (1707-1783) and Joseph-Louis Lagrange (1736-1813). The equations of motion are therefore called Euler-Lagrange equations. William Rowan Hamilton (1805-1865) later brought it into a form that is now very useful for numerical solution on the computer.

When it comes to systems of extended solids, be they rigid or deformable bodies, the newer finite element method is used. This is very computationally intensive. It has been used very successfully in engineering since the advent of modern computers.

As part of an attempt to explain a possible excess energy in the Milkovic pendulum, an anonymous author carried out a Lagrangian calculation [3]. To the best of our knowledge, this is the most in-depth analysis of this system. We checked the calculation, but found a serious modeling error by the author. That makes his results, which actually show an energy surplus, quite questionable. As a second system, which is based on the principle of the double pendulum, we examined the planetary gear according to Würth [2]. These results will be reported in a subsequent paper. This is probably the first time that such considerations have been made at this modeling depth. In the following, we first briefly describe the application class of the double pendulum and the Lagrange method before we go into the results of the double pendulum.

2 Calculation method of the double pendulum

A double pendulum consists of two pendulums that are attached to one another and each have a pendulum body with a mass. When impacted in a vertical position, these masses perform unpredictable oscillations, it is a well-known example of a chaotic system. According to the Lagrange theory, the description requires coordinates that correspond to the number of degrees of freedom. Here it is the angular deflections φ_1 and φ_2 from the vertical, see Fig. 1. To calculate the Lagrangian

$$\mathscr{L} = T - U \tag{1}$$

one needs the kinetic energy T and the potential energy U of the two masses. The easiest way to get the kinetic energy is from the Cartesian coordinates. It is according to Fig. 1:

$$x_1 = l_1 \sin(\varphi_1), \tag{2}$$
$$y_1 = -l_1 \cos(\varphi_1), \tag{3}$$

and

$$x_2 = x_1 + l_2 \sin(\varphi_2), \tag{4}$$
$$y_2 = y_1 - l_2 \cos(\varphi_2). \tag{5}$$

This gives the kinetic energy of the masses m_1 and m_2:

$$T_1 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2), \tag{6}$$

$$T_2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2), \tag{7}$$

$$T = T_1 + T_2. \tag{8}$$

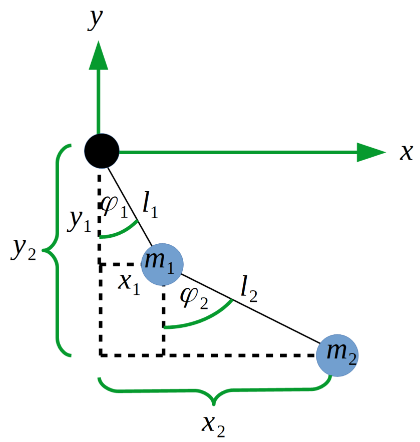

Figure 1: Coordinates of the double pendulum.

The point denotes the time derivatives. The potential energy follows from the force of gravity in the y direction:

$$U = m_1 g\, y_1 + m_2 g\, y_2 \tag{9}$$

with the gravitational acceleration g. The Lagrange function (1) is thus completely determined. The equations of motion follow from the Euler-Lagrange equations, where q_i stands for the coordinates φ_1 and φ_2:

$$\frac{d}{dt} \frac{\partial \mathscr{L}}{\partial \dot{q}_i} - \frac{\partial \mathscr{L}}{\partial q_i} = 0. \tag{10}$$

Conservation of energy applies to these equations, because they are derived from there. In addition, one can introduce so-called dissipation functions D_i and generalized forces Q_i. A generalized force in our case is a torque. Then the equations take the form:

$$\frac{d}{dt} \frac{\partial \mathscr{L}}{\partial \dot{q}_i} - \frac{\partial \mathscr{L}}{\partial q_i} + \frac{\partial D_i}{\partial \dot{q}_i} = Q_i \tag{11}$$

and there is no longer any conservation of energy. We need this case here, since the system is supposed to provide additional energy. The dissipation functions can be replaced by

$$Q_{Ri} = -\frac{\partial D_i}{\partial \dot{q}_i} \tag{12}$$

and be traced back to generalized forces, with Q_{Ri} denoting any residual forces that are not covered by the dissipation function.

The evaluation of equations (10) can lead to very complicated equations of motion. In the case of the double pendulum, they are just about manageable. In other cases, such as the planetary gear, they are so complicated that it is no longer possible to calculate them by hand. We use the computer algebra system Maxima for this. The Euler-Lagrange equations are 2 coupled differential equations for the variables $\ddot{\varphi}_1$ and $\ddot{\varphi}_2$. They are linear in these two variables and have to be solved for these so that the numerical solution (time integration) can be carried out. The following equations then result:

$$\begin{aligned}\ddot{\varphi}_1 = \Big[& \left(l_1\, l_2\, m_2 \sin\left(\varphi_2\right) \cos\left(\varphi_2 - \varphi_1\right) - \left(l_1\, l_2\, m_2 + l_1\, m_1\, l_2\right) \sin\left(\varphi_1\right)\right) g \\ & + \left(l_1^2\, l_2\, m_2\, \dot{\varphi}_1^2 \cos\left(\varphi_2 - \varphi_1\right) + l_1\, l_2^2\, m_2\, \dot{\varphi}_2^2\right) \sin\left(\varphi_2 - \varphi_1\right) \\ & - l_1\, Q_2 \cos\left(\varphi_2 - \varphi_1\right) + l_2\, Q_1 \Big] \\ & \cdot \frac{1}{l_1^2\, l_2\, m_2 \sin\left(\varphi_2 - \varphi_1\right)^2 + l_1^2\, l_2\, m_1}, \end{aligned} \tag{13}$$

$$\begin{aligned}\ddot{\varphi}_2 = \Big[& \left(\left(l_1\, l_2\, m_2^2 + l_1\, m_1\, l_2\, m_2\right) \sin\left(\varphi_1\right) \cos\left(\varphi_2 - \varphi_1\right) - \left(l_1\, l_2\, m_2^2 + l_1\, m_1\, l_2\, m_2\right) \sin\left(\varphi_2\right)\right) g \\ & + \left(\left(-l_1^2\, l_2\, m_2^2 - l_1^2\, m_1\, l_2\, m_2\right)\, \dot{\varphi}_1^2 - l_1\, l_2^2\, m_2^2\, \dot{\varphi}_2^2 \cos\left(\varphi_2 - \varphi_1\right)\right) \sin\left(\varphi_2 - \varphi_1\right) \\ & - Q_1\, l_2\, m_2 \cos\left(\varphi_2 - \varphi_1\right) + l_1\, Q_2\, m_2 + l_1\, m_1\, Q_2 \Big] \\ & \cdot \frac{1}{l_1\, l_2^2\, m_2^2 \sin\left(\varphi_2 - \varphi_1\right)^2 + l_1\, l_2^2\, m_1\, m_2}. \end{aligned} \tag{14}$$

3 Results

3.1 Verification and comparison with [3]

The aim of our calculations was initially to verify the results of the work [3]. We first compared the equations of motion cited in the reference work with ours. There was no complete match. The anonymous author has not given the source of his equations and only speaks of "literature". Since this literature comes with certainty from the time when there was no computer algebra, it cannot be ruled out that there is a calculation error. It wouldn't be the first time such errors have been found in textbooks.

For comparison, we used the same parameters for the double pendulum as the anonymous author, see Table 1. Here $\omega_{1,2}$ is to be equated with the angular velocity $\dot{\varphi}_{1,2}$. The calculation was initially carried out without external forces and without gravitation. The initial angular velocity of the second pendulum is 50 Hz, so it is quite fast. In his Fig. 2, in addition to the time course for ω_1, the author also gives a "Pivot Torque" and a "Pivot Power", i.e., a torque and a power on the fixed axis, which he calculates as follows:

$$\tau_{\text{pivot torque}} = m_1 l_1^2 \ddot{\varphi}_1, \tag{15}$$

$$P_{\text{pivot power}} = m_1 l_1^2 \ddot{\varphi}_1 \omega_1. \tag{16}$$

m_1	1 kg
m_2	0.1 kg
l_1	0.2 m
l_2	0.1 m
$\varphi_{1,\text{initial}}$	0
$\varphi_{2,\text{initial}}$	0
$\omega_{1,\text{initial}}$	0
$\omega_{2,\text{initial}}$	$100{\cdot}\pi$ rad/s

Table 1: Parameters and initial values for calculating the double pendulum.

This is where the accelerations on the axis appear. We have also evaluated these variables and shown them in Fig. 2. The angular velocity ω_2 results from the initial values and is exactly the same as in Fig. 2 of the reference document. Torque and power are a few percent lower, but otherwise the same. That may be an influence of the different equations of motion. Each of torque and power cancel each on average over time. The author speaks of reactive power. This has to be the case, since no power is taken out of the system.

Next, we look at the energy balance. Since we have neither external forces nor gravity, there are only kinetic energy contributions that have to be constant in sum for both masses. This is the case, as can be seen from Fig. 3. The sum corresponds to the initial rotation of the second mass of 50 Hz, which is a little over 49 joules. The same curve is found as Figure 3 in the reference document. So far, there is agreement.

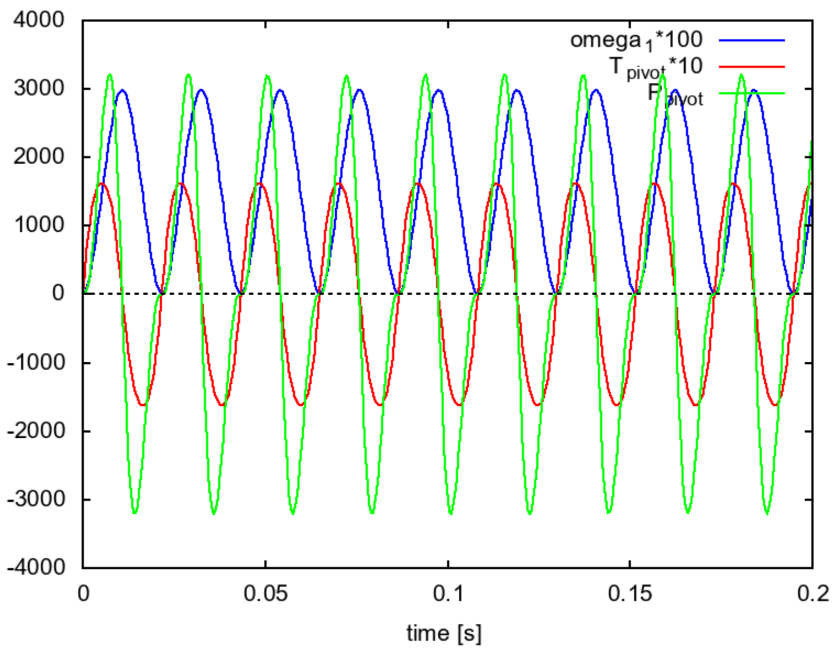

Figure 2: Angular velocity [rad/s], Pivot torque [Nm] und Pivot power [W].

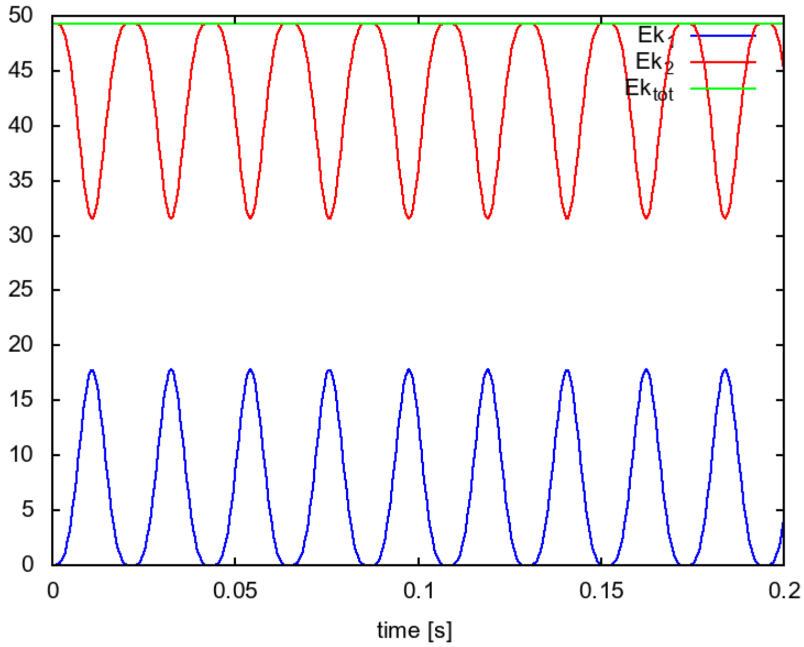

Figure 3: Kinetic energies [J] of both masses and total energy.

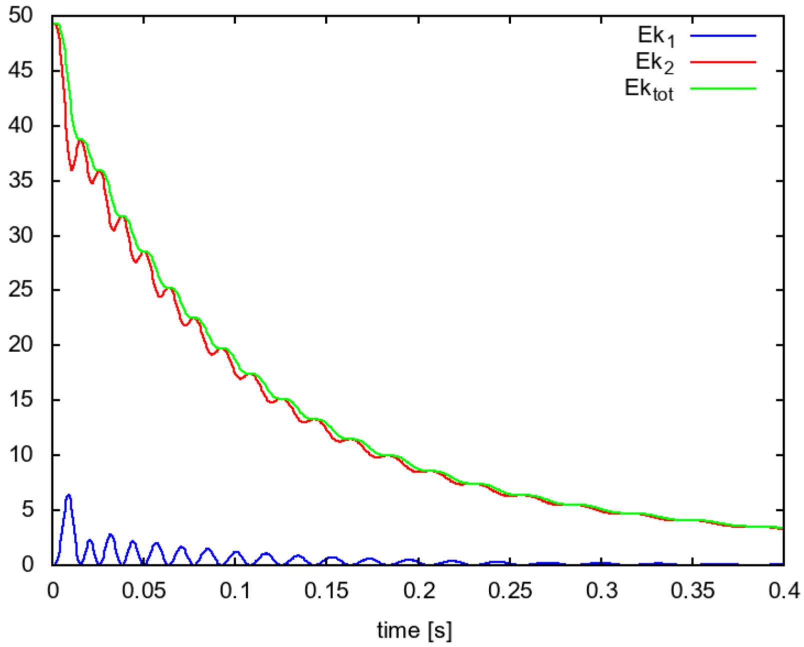

Figure 4: Kinetic energies [J] of both masses and total energy for external load, canonical form.

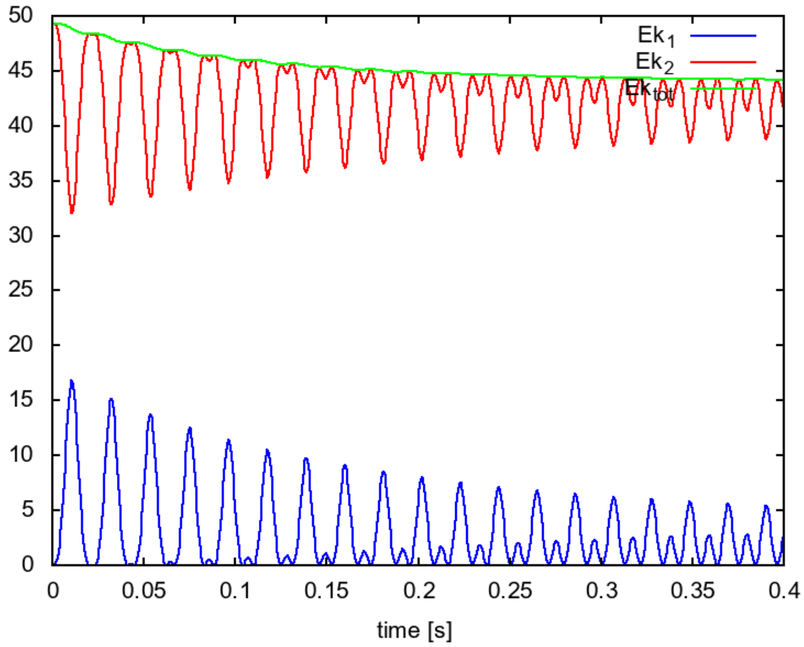

Figure 5: Kinetic energies [J] of both masses and total energy for external load. afterh [3].

We now apply an external load (a braking torque) to the fixed axis. This is modeled as a generalized force in the form

$$Q_1 = -\frac{\mu}{l_1}\omega_1, \tag{17}$$

where μ is a constant. We do this based on the work [3]. The braking torque is proportional to the angular velocity. The calculation with $\mu = 1$ shows that the total energy decreases exponentially, see Fig. 4. At the same time, the frequency slows down. The rotational energy of the outer, fast pendulum is transferred to the inner pendulum, from where it is removed from the system by the load torque. The fact that the braking torque, which only acts on the axis of the first pendulum, also acts on the outer pendulum, can be seen directly from the equation of motion (14). In addition to Q_2 (not used here), this equation also contains the braking torque Q_1.

The anonymous author of [3] received completely different results for the braking torque (14). He did not use the concept of generalized force as prescribed by the Lagrange theory, but changed equation (13) for the acceleration of φ_1 a posteriori by making the replacement:

$$\ddot{\varphi}_1 \rightarrow \ddot{\varphi}_1 + \frac{\mu}{l_1}\omega_1. \tag{18}$$

Here the parameter μ has different physical units than in (17), but that is not decisive. As a result of this arbitrary substitution, the braking torque only acts on the movement of the inner pendulum and the rotation of the outer pendulum is not affected. Our calculation with this approach results in the energy curve of Fig. 5, which is identical to that of Fig. 6 in [3]. The total energy only decreases slightly at the beginning and then remains constant. Only the form of oscillation changes; the oscillation frequencies are doubled due to the external braking torque. As we have explained, this is an arbitrary intervention in the “physics” of the double pendulum. Therefore, all results in [3] based on this are unfortunately to be regarded as unphysical.

3.2 Effect of different load torques

We can investigate how the load torque has to be changed so that there may be an increase in energy, if one uses the correct equations of motion. With the approach

$$Q_1 = -\frac{\mu}{l_1}|\omega_1| \tag{19}$$

(modulus of ω_1) and $\mu = 0.1$ the result of Fig. 6 come out. A phase change occurs for both pendulums, whereby – after an initial decrease in the total energy – there is a gain. The question is whether this corresponds to an energy gain in the overall system or whether this increase is due to the supply of external energy. To this end, we examine the external torque or braking torque τ_{ext} and the input or output power P_{ext}. The following applies:

$$\tau_{\text{ext}} = Q_1 = -\frac{\mu}{l_1}|\omega_1|, \tag{20}$$

$$P_{\text{ext}} = \tau_{\text{ext}}\omega_1. \tag{21}$$

Both are shown in Fig. 7. The torque, based on the modulus in Eq. (20), is always negative. For the phases in which the angular velocity ω_1 is also negative, this leads to a torque of the same sign, i.e. the mass m_1 is driven in this direction and the angle even reverses the direction. Accordingly, the performance (21) is positive, i.e. energy is supplied, as can be seen from Fig. 7. So this is a drive effect, the system does not provide any energy gain.

One can avoid the drive phase by only using real braking phases for the braking torque:

$$\tau_{\text{ext}} = Q_1 = \begin{cases} -\frac{\mu}{l_1}\omega_1 & \text{f'ur } \omega_1 > 0, \\ 0 & \text{else.} \end{cases} \tag{22}$$

Then the total energy adjusts to a final value after an initial braking phase, as shown in Fig. 8 for $\mu = 0.1$. The external energy flow can be determined via the integral

$$E_{\text{ext}} = \int P_{\text{ext}} dt. \tag{23}$$

The numerical evaluation in Fig. 9 shows that initially energy actually flows away (negative values), but then the energy remains constant, i.e., the drain has ”dried up”. So there is no energy gain to be drawn from the system itself in this way.

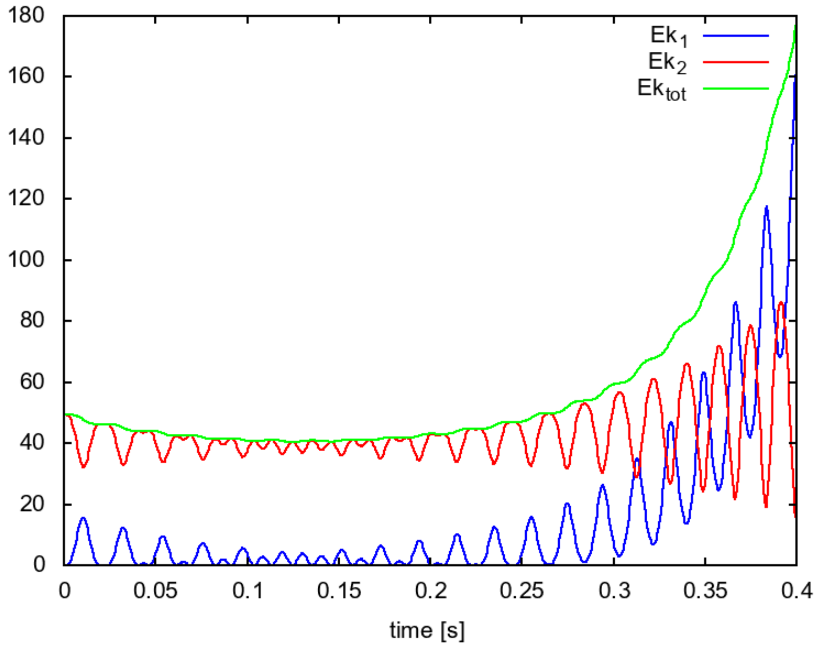

Figure 6: Kinetic energies of both masses and total energy for load (19).

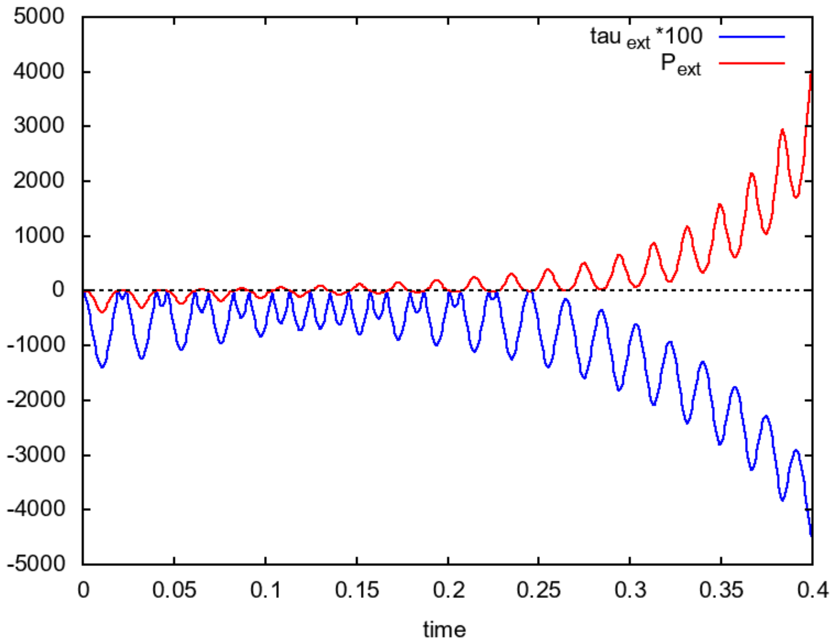

Figure 7: External load torque and power for load (19).

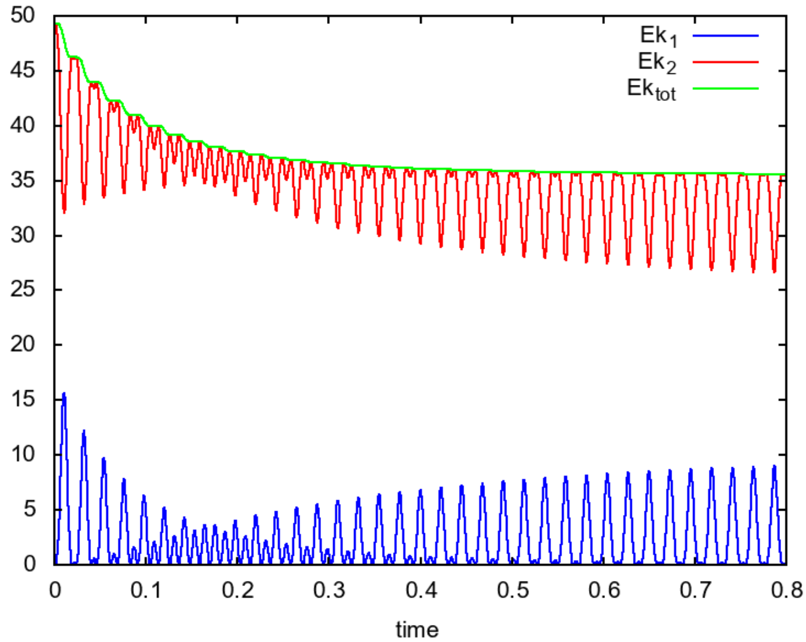

Figure 8: Kinetic energies of both masses and total energy for load (20).

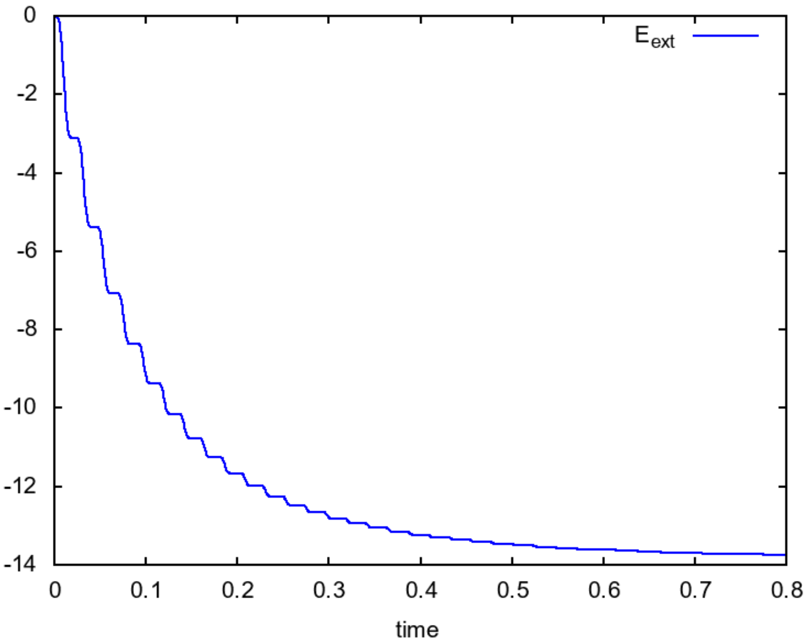

Figure 9: Externally released energy for load (20).

3.3 Effect of interaction with "space energy"

To get the desired effect of energy gain we have to consider mechanisms that cannot be found in conventional physics. We assume a resonance mechanism for this, which is predicted by the ECE theory [4–6]. This mechanism was calculated for electromagnetic systems, but due to the complete equivalence between electromagnetic and mechanical systems, it also applies to dynamics. According to Eq. (20) in [6], the resonance equation applies there

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} + \omega_t \frac{\partial \mathbf{A}}{\partial t} + \frac{\partial \omega_t}{\partial t} \mathbf{A} = \frac{1}{\epsilon_0} \mathbf{J}, \tag{24}$$

where $\mathbf{A}$ is the vector potential, $\mathbf{J}$ is the electrical current density and ω_t is the spin connection (a frequency) of Cartan geometry. Applied to mechanics, this equation reads:

$$\frac{\partial^2 \mathbf{Q}}{\partial t^2} + \omega_t \frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \omega_t}{\partial t} \mathbf{Q} = G\, \mathbf{J}_m, \tag{25}$$

where $\mathbf{Q}$ is the mechanical equivalent of the vector potential, $\mathbf{J}_m$ is the mass flow density and G is the Einstein constant. $\mathbf{Q}$ has the units of a velocity, one can consider it as "aether flow". If we restrict ourselves to the rotation component (the φ componentQ_φ) of $\mathbf{Q}$ and assume a linear time dependence of ω_t, this equation can be written as:

$$\frac{d^2 Q_\varphi}{dt^2} + \alpha \frac{dQ_\varphi}{dt} + \omega_0^2 Q_\varphi = G\, J_\varphi, \tag{26}$$

and with a periodic excitaiton $J_\varphi = G\, J_0 \cos(\omega t)$:

$$\frac{d^2 Q_\varphi}{dt^2} + \alpha \frac{dQ_\varphi}{dt} + \omega_0^2 Q_\varphi = G\, J_0 \cos(\omega t). \tag{27}$$

This is the equation of a damped resonance with a resonance frequency ω_0 and a damping constant α. The solution to this differential equation is:

$$Q_\varphi = G\, J_0 \frac{\alpha\omega \sin(\omega t) + \left({\omega_0}^2 - \omega^2\right)\cos(\omega t)}{(\omega_0^2 - \omega^2)^2 + \alpha^2\omega^2}. \tag{28}$$

For $\alpha \approx 0$ the solution is simplified to

$$Q_\varphi = G\, J_0 \frac{\cos(\omega t)}{\omega_0^2 - \omega^2}, \tag{29}$$

which means a resonance increase of Q_φ of infinite, i.e. arbitrarily high strength. We now apply this to the double pendulum. We assume that the outer pendulum rotates relatively quickly, as assumed in the previous calculations. Then it makes sense to assume an energy transfer due to the rotational potential Q_φ. In the Lagrange formalism, this then appears as the external torque Q_2:

$$Q_2 = Q_\varphi = G\, J_0 \frac{\alpha\omega_2 \sin(\omega_2 t) + \left({\omega_0}^2 - {\omega_2}^2\right)\cos(\omega_2 t)}{(\omega_0^2 - \omega_2^2)^2 + \alpha^2\omega_2^2}. \tag{30}$$

We have set $\omega = \omega_2$, the angular velocity of the outer pendulum. So that an influence of Q_2 becomes visible, we have to place the initial value of ω_2 close to the resonance frequency ω_0. In addition, we assume a decrease in energy due to deceleration on the central axis of rotation, as previously applied by Eq. (17):

$$Q_1 = -\frac{\mu}{l_1}\omega_1. \tag{31}$$

The new constants and initial values are listed in Table 2, resulting in the kinetic energy curve shown in Fig. 10. The resonance structure of Q_2 creates chaotic behavior in parts, which makes numerical stability of the result difficult. However, the solution shown could be reproduced qualitatively when the time integration step size Δt was varied. In Fig. 10, the kinetic energy calculated from the initial conditions is also shown. You can see that the resonance provides a significant amount of additional energy, except in an initial transient range.

μ	0.05
GJ_0	50 000
α	5.0
ω_0	$11\cdot\pi$
$\omega_{2,\text{initial}}$	$10\cdot\pi$

Table 2: Parameters and modified initial values for calculating the space energy effect.

Since we have taken the braking force into account, the energy increase takes place with the release of useful energy. This was calculated according to Eq. (23) (as for Fig. 9) and is shown in Fig. 11. After a settling phase, an approximately constant amount of energy per time unit is emitted, i.e., we can withdraw a constant continuous output from the system. We have thus found a possible mechanism for a double pendulum that is fed by space energy, provided that the prediction made by the ECE theory actually applies and can be demonstrated in the experiment.

4 Summary

The extraction of power from a system with two coupled, vibrating units, claimed by Milkovic and the anonymous author, could not be confirmed in this study. Different approaches to power extraction always lead to a decrease in rotational energy, i.e., a conservation of the total energy. This is no different to be expected from classic rotating systems. Such a system can only generate energy from itself, if it is in exchange with an external energy reservoir. Such a reservoir is the space energy of the non-empty vacuum. The model calculation has shown that rotary fields of the vacuum can provide such an effect. Even if some inventors claim to have found such an energy source, scientific verification is still pending. This includes the reproducibility and repeatability of such experiments.

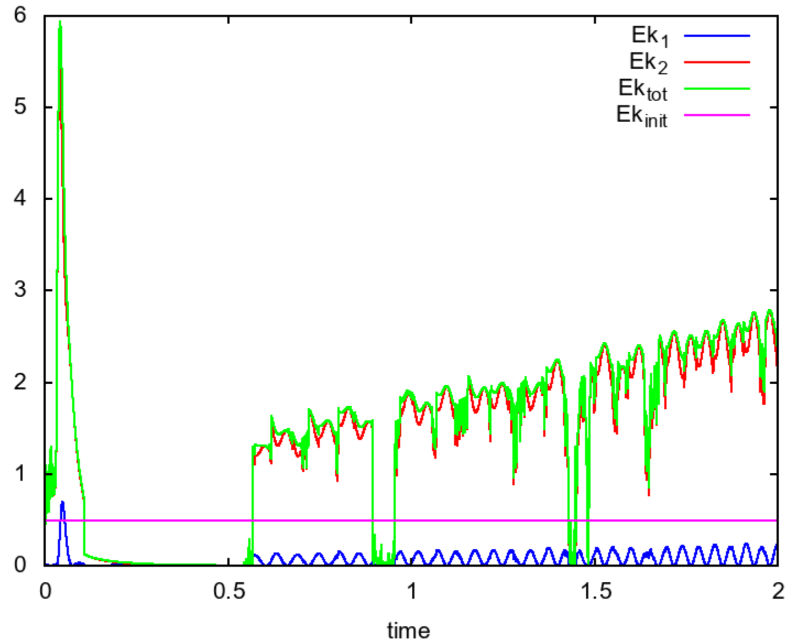

Figure 10: Kinetic energies of both masses, total energy and initial energy Ek_{init} with spatce energy coupling.

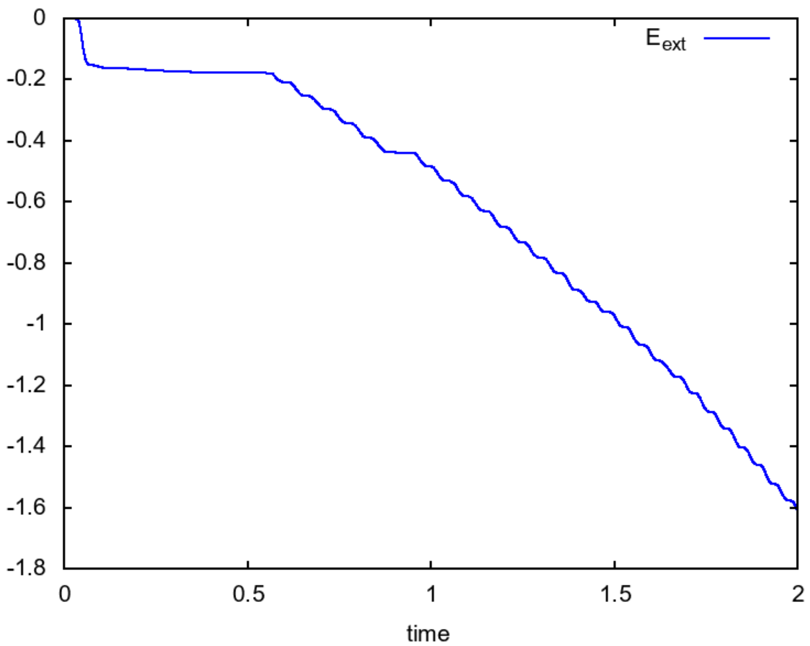

Figure 11: Externally emitted energy with space energy coupling.

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