```
(%i1)  kill(all);
(%o0)  done
```

```
(%i1)  depends([p1,q1,p2,q2,r1],t);
(%o1)  [p1(t), q1(t), p2(t), q2(t), r1(t)]
```

1 Eq. (15)

```
(%i2)  E2: m(r1)*(p1^2*c^2+m^2*c^4);
```

$$(c^2\,p1^2+c^4\,m^2)\,\mathrm{m}(r1)$$

```
(%i3)  E: sqrt(E2);
```

$$\sqrt{c^2\,p1^2+c^4\,m^2}\,\sqrt{\mathrm{m}(r1)}$$

```
(%i4)  diff(E, r1);
```

$$\frac{\sqrt{c^2\,p1^2+c^4\,m^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,r1}\,\mathrm{m}(r1)\right)}{2\,\sqrt{\mathrm{m}(r1)}}$$

```
(%i5)  diff(m(r1)^(1/2), r1);
```

$$\frac{\frac{\mathrm{d}}{\mathrm{d}\,r1}\,\mathrm{m}(r1)}{2\,\sqrt{\mathrm{m}(r1)}}$$

2 Hamilton equations

```
(%i7)  H1(H,q,p) := diff(q,t) = diff(H, p);
       H2(H,q,p) := diff(p,t) = -diff(H, q);
```

$$\mathrm{H1}(H,q,p) := \frac{\mathrm{d}}{\mathrm{d}\,t}\,q = \frac{\mathrm{d}}{\mathrm{d}\,p}\,H$$

$$\mathrm{H2}(H,q,p) := \frac{\mathrm{d}}{\mathrm{d}\,t}\,p = -\frac{\mathrm{d}}{\mathrm{d}\,q}\,H$$

3 Eval Hamilton equations (inertial system)

```
(%i8)  H: sqrt(c^2*p1^2+c^4*m^2)*sqrt(m(q1))-m*M*G/q1;
```

$$\sqrt{c^2\,p1^2+c^4\,m^2}\,\sqrt{\mathrm{m}(q1)}-\frac{G\,M\,m}{q1}$$

```
(%i12)  E21: H1(H,q1,p1);
        E11: H1(H,q2,p2);
        E31: H2(H,q1,p1);
        E41: H2(H,q2,p2);
```

$$(\%o9)\quad \frac{d}{dt}\,q1=\frac{c^2\,p1\,\sqrt{m(q1)}}{\sqrt{c^2\,p1^2+c^4\,m^2}}$$

$$(\%o10)\quad \frac{d}{dt}\,q2=0$$

$$(\%o11)\quad \frac{d}{dt}\,p1=-\frac{\sqrt{c^2\,p1^2+c^4\,m^2}\left(\frac{d}{dq1}\,m(q1)\right)}{2\sqrt{m(q1)}}-\frac{G\,M\,m}{q1^2}$$

$$(\%o12)\quad \frac{d}{dt}\,p2=0$$

4 Eval Hamilton equations (polar coordinate syste

```
(%i13)  H: sqrt(c^2*(p1^2+p2^2/q1^2)+c^4*m^2)*sqrt(m(q1))-(G*M*m)
```

$$(\%o13)\quad \sqrt{c^2\left(\frac{p2^2}{q1^2}+p1^2\right)+c^4\,m^2}\,\sqrt{m(q1)}-\frac{G\,M\,m}{q1}$$

```
(%i17)  E11: H1(H,q1,p1);
        E21: H1(H,q2,p2);
        E31: H2(H,q1,p1);
        E41: H2(H,q2,p2);
```

$$(\%o14)\quad \frac{d}{dt}\,q1=\frac{c^2\,p1\,\sqrt{m(q1)}}{\sqrt{c^2\left(\frac{p2^2}{q1^2}+p1^2\right)+c^4\,m^2}}$$

$$(\%o15)\quad \frac{d}{dt}\,q2=\frac{c^2\,p2\,\sqrt{m(q1)}}{\sqrt{c^2\left(\frac{p2^2}{q1^2}+p1^2\right)+c^4\,m^2\,q1^2}}$$

$$(\%o16)\quad \frac{d}{dt}\,p1=-\frac{\sqrt{c^2\left(\frac{p2^2}{q1^2}+p1^2\right)+c^4\,m^2}\left(\frac{d}{dq1}\,m(q1)\right)}{2\sqrt{m(q1)}}+\frac{c^2\,p2^2\,\sqrt{m(q1)}}{\sqrt{c^2\left(\frac{p2^2}{q1^2}+p1^2\right)+c^4\,m^2}\,q1^3}-\frac{G\,M\,m}{q1^2}$$

$$(\%o17)\quad \frac{d}{dt}\,p2=0$$

4.1 Insert epsilon_1

```
(%i18) fac: 1/sqrt(c^2*(p2^2/q1^2+p1^2)+c^4*m^2);
```

(%o18)
$$\frac{1}{\sqrt{c^2\left(\frac{p2^2}{q1^2}+p1^2\right)+c^4\,m^2}}$$

```
(%i22) E11a: ratsubst(epsilon_1, fac, E11);
       E21a: ratsubst(epsilon_1, fac, E21);
       E31a: expand(ratsubst(epsilon_1, fac, E31));
       E41a: ratsubst(epsilon_1, fac, E41);
```

(%o19)
$$\frac{\mathrm{d}}{\mathrm{d}\,t}\,q1=c^2\,\epsilon_1\,p1\,\sqrt{\mathrm{m}(q1)}$$

(%o20)
$$\frac{\mathrm{d}}{\mathrm{d}\,t}\,q2=\frac{c^2\,\epsilon_1\,p2\,\sqrt{\mathrm{m}(q1)}}{q1^2}$$

(%o21)
$$\frac{\mathrm{d}}{\mathrm{d}\,t}\,p1=-\frac{\frac{\mathrm{d}}{\mathrm{d}\,q1}\,\mathrm{m}(q1)}{2\,\epsilon_1\,\sqrt{\mathrm{m}(q1)}}+\frac{c^2\,\epsilon_1\,p2^2\,\sqrt{\mathrm{m}(q1)}}{q1^3}-\frac{G\,M\,m}{q1^2}$$

(%o22)
$$\frac{\mathrm{d}}{\mathrm{d}\,t}\,p2=0$$

5 Hamilton-Jacobi equation of m theory

```
(%i23) kill(all);
(%o0)  done
```

```
(%i1)  assume(r>0, L>0, m>0, M>0, G>0, c>0);
(%o1)  [r>0,L>0,m>0,M>0,G>0,c>0]
```

```
(%i2)  Ea: E = (m(r)*(c^2*(diff(S_r(r),r)^2+L^2/r^2)+m^2*c^4))^(
```

(%o2)
$$E=\sqrt{\mathrm{m}(r)}\,\sqrt{c^2\left(\left(\frac{\mathrm{d}}{\mathrm{d}\,r}\,S_r(r)\right)^2+\frac{L^2}{r^2}\right)+c^4\,m^2}-\frac{G\,M\,m}{r}$$

```
(%i3)  Eb: expand((Ea+(G*M*m)/r)^2);
```

(%o3)
$$\frac{2\,E\,G\,M\,m}{r}+\frac{G^2\,M^2\,m^2}{r^2}+E^2=c^2\,\mathrm{m}(r)\left(\frac{\mathrm{d}}{\mathrm{d}\,r}\,S_r(r)\right)^2+\frac{L^2\,c^2\,\mathrm{m}(r)}{r^2}+c^4\,m^2\,\mathrm{m}(r)$$

(%i4) `Ec: Eb-(L^2*c^2*m(r)/r^2+c^4*m^2*m(r));`

(%o4)
$$-\frac{L^2\,c^2\,\mathrm{m}(r)}{r^2}-c^4\,m^2\,\mathrm{m}(r)+\frac{2\,E\,G\,M\,m}{r}+\frac{G^2\,M^2\,m^2}{r^2}+E^2=c^2\,\mathrm{m}(r)\left(\frac{\mathrm{d}}{\mathrm{d}\,r}\,S_r(r)\right)^2$$

(%i5) `Ed: Ec/(c^2*m(r));`

(%o5)
$$\frac{-\frac{L^2\,c^2\,\mathrm{m}(r)}{r^2}-c^4\,m^2\,\mathrm{m}(r)+\frac{2\,E\,G\,M\,m}{r}+\frac{G^2\,M^2\,m^2}{r^2}+E^2}{c^2\,\mathrm{m}(r)}=\left(\frac{\mathrm{d}}{\mathrm{d}\,r}\,S_r(r)\right)^2$$

(%i6) `Ee: sqrt(Ed);`

(%o6)
$$\frac{\sqrt{\frac{-\frac{L^2\,c^2\,\mathrm{m}(r)}{r^2}-c^4\,m^2\,\mathrm{m}(r)+\frac{2\,E\,G\,M\,m}{r}+\frac{G^2\,M^2\,m^2}{r^2}+E^2}{\mathrm{m}(r)}}}{c}=\left|\frac{\mathrm{d}}{\mathrm{d}\,r}\,S_r(r)\right|$$

(%i7) `Ef: diff(S_r(r),r) = radcan(lhs(Ee));`

(%o7)
$$\frac{\mathrm{d}}{\mathrm{d}\,r}\,S_r(r)=\frac{\sqrt{\left(-c^4\,m^2\,r^2-L^2\,c^2\right)\mathrm{m}(r)+E^2\,r^2+2\,E\,G\,M\,m\,r+G^2\,M^2\,m^2}}{c\,r\,\sqrt{\mathrm{m}(r)}}$$

(%i8) `Eg: ode2(Ef,S_r(r),r);`

(%o8)
$$S_r(r)=\frac{\int\frac{\sqrt{\left(-c^4\,m^2\,r^2-L^2\,c^2\right)\mathrm{m}(r)+E^2\,r^2+2\,E\,G\,M\,m\,r+G^2\,M^2\,m^2}}{r\,\sqrt{\mathrm{m}(r)}}\,\mathrm{d}\,r}{c}+\%c$$

5.1 Solution with m(r)=1

(%i9) `m(r):=1;`

(%o9) $\mathrm{m}(r):=1$

```
(%i10)  str: [E=-7.6+400, L=2.215, m=1, M=10, G=1, c=20];
```

$$(\%o10)\quad [E=392.4, L=2.215, m=1, M=10, G=1, c=20]$$

```
(%i12)  Eh[1]: ev(Ef, str);
        Eh[2]: ev(lhs(Ef)=-rhs(Ef), str);
```

$$(\%o11)\quad \frac{d}{d\,r}\,S_r(r)=\frac{\sqrt{-6022.24000000002\,r^2+7848.0\,r-1862.49}}{20\,r}$$

$$(\%o12)\quad \frac{d}{d\,r}\,S_r(r)=-\frac{\sqrt{-6022.24000000002\,r^2+7848.0\,r-1862.49}}{20\,r}$$

```
(%i14)  Ei[1]: ode2(Eh[1],S_r(r),r)-%c;
        Ei[2]: ode2(Eh[2],S_r(r),r)-%c;
rat: replaced -1862.49 by -186249/100 = -1862.49
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -6022.24000000002 by -12171910020265/2021159904 = -
rat: replaced -1862.49 by -186249/100 = -1862.49
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -6022.24000000002 by -12171910020265/2021159904 = -
```

(%o13)

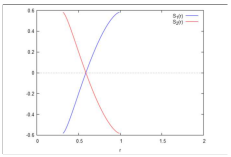
$$S_r(r)-\%c=\left(\sqrt{-304297750506625\,r^2+396551573164800\,r-94109752740024}-\frac{39655157316480\,\operatorname{asin}\left(\frac{396551573164800-608595501013250\,r}{60\,\sqrt{11862112761494933604527890}}\right)}{\sqrt{12171910020265}}+42\right)\sqrt{53350199966}\,\operatorname{asin}\left(\frac{15684958790004}{5\,\sqrt{11862112761494933604527890}\,r}-\frac{6609192886080}{\sqrt{11862112761494933604527890}}\right)/\left(400\,\sqrt{126322494}\right)$$

```
rat: replaced -1862.49 by -186249/100 = -1862.49
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -6022.24000000002 by -12171910020265/2021159904 = -
rat: replaced -1862.49 by -186249/100 = -1862.49
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -6022.24000000002 by -12171910020265/2021159904 = -
```

(%o14)

$$S_r(r)-\%c=-\left(\sqrt{-304297750506625\,r^2+396551573164800\,r-94109752740024}-\frac{39655157316480\,\operatorname{asin}\left(\frac{396551573164800-608595501013250\,r}{60\,\sqrt{11862112761494933604527890}}\right)}{\sqrt{12171910020265}}+42\right)\sqrt{53350199966}\,\operatorname{asin}\left(\frac{15684958790004}{5\,\sqrt{11862112761494933604527890}\,r}-\frac{6609192886080}{\sqrt{11862112761494933604527890}}\right)/\left(400\,\sqrt{126322494}\right)$$

```
(%i15)  wxplot2d([rhs(Ei[1]),rhs(Ei[2])], [r,0,2],
        [ legend, "S_1(r)", "S_2(r)", "S_3(r)", "S_4(r)"])$
plot2d: expression evaluates to non-numeric value somewhere in pl
plot2d: expression evaluates to non-numeric value somewhere in pl
```



```
(%t15)
```

5.2 Solution with m(r)=0.99

```
(%i16)  m(r):=.99;
```

$$(\%o16)\quad m(r):=0.99$$

```
(%i17)  str: [E=-7.6+400, L=2.215, m=1, M=10, G=1, c=20];
```

$$(\%o17)\quad [E=392.4, L=2.215, m=1, M=10, G=1, c=20]$$

```
(%i19)  Eh[1]: ev(Ef, str);
        Eh[2]: ev(lhs(Ef)=-rhs(Ef), str);
```

(%o18)
$$\frac{d}{d\,r}\,S_r(r)=\frac{0.0502518907629606\,\sqrt{153977.76\,r^2+0.99\left(-160000\,r^2-1962.49\right)+7848.0\,r+100}}{r}$$

(%o19)
$$\frac{d}{d\,r}\,S_r(r)=-\frac{0.0502518907629606\,\sqrt{153977.76\,r^2+0.99\left(-160000\,r^2-1962.49\right)+7848.0\,r+100}}{r}$$

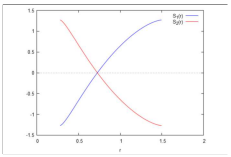
```
(%i21)   Ei[3]: ode2(Eh[1],S_r(r),r)-%c;
         Ei[4]: ode2(Eh[2],S_r(r),r)-%c;
rat: replaced -0.0502518907629606 by -1584030/31521799 = -0.05025
rat: replaced -1842.8651 by -18428651/10000 = -1842.8651
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -4422.24000000002 by -8938054173865/2021159904 = -4
rat: replaced -0.0502518907629606 by -1584030/31521799 = -0.05025
rat: replaced -1842.8651 by -18428651/10000 = -1842.8651
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -4422.24000000002 by -8938054173865/2021159904 = -4
rat: replaced -0.0502518907629606 by -1584030/31521799 = -0.05025
```

$$(\%o20)\quad S_r(r)-\%c=(158403\,(\sqrt{-5586283858665625\,r^2+9913789329120000\,r-2327953155375594}-\frac{198275786582400\,\operatorname{asin}\left(\frac{9913789329120000-11172567717331250\,r}{50\,\sqrt{1850591612790240705853610919 0}}\right)}{\sqrt{8938054173865}}+\sqrt{2327953155375594}\,\operatorname{asin}\left(\frac{2327953155375594}{25\,\sqrt{1850591612790240705853610919 0}\,r}-\frac{198275786582400}{\sqrt{1850591612790240705853610919 0}}\right)))/(315217990\,\sqrt{126322494})$$

```
rat: replaced 0.0502518907629606 by 1584030/31521799 = 0.0502518
rat: replaced -1842.8651 by -18428651/10000 = -1842.8651
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -4422.24000000002 by -8938054173865/2021159904 = -4
rat: replaced 0.0502518907629606 by 1584030/31521799 = 0.0502518
rat: replaced -1842.8651 by -18428651/10000 = -1842.8651
rat: replaced 7848.0 by 7848/1 = 7848.0
rat: replaced -4422.24000000002 by -8938054173865/2021159904 = -4
rat: replaced 0.0502518907629606 by 1584030/31521799 = 0.0502518
```

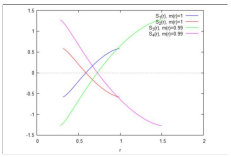
$$(\%o21)\quad S_r(r)-\%c=-(158403\,(\sqrt{-5586283858665625\,r^2+9913789329120000\,r-2327953155375594}-\frac{198275786582400\,\operatorname{asin}\left(\frac{9913789329120000-11172567717331250\,r}{50\,\sqrt{1850591612790240705853610919 0}}\right)}{\sqrt{8938054173865}}+\sqrt{2327953155375594}\,\operatorname{asin}\left(\frac{2327953155375594}{25\,\sqrt{1850591612790240705853610919 0}\,r}-\frac{198275786582400}{\sqrt{1850591612790240705853610919 0}}\right)))/(315217990\,\sqrt{126322494})$$

```
(%i22)  wxplot2d([rhs(Ei[3]),rhs(Ei[4])], [r,0,2],
        [ legend, "S_1(r)", "S_2(r)", "S_3(r)", "S_4(r)"])$
plot2d: expression evaluates to non-numeric value somewhere in pl
plot2d: expression evaluates to non-numeric value somewhere in pl
```



```
(%t22)
```

```
(%i23)  wxplot2d([rhs(Ei[1]),rhs(Ei[2]),rhs(Ei[3]),rhs(Ei[4])], [
        [ legend, "S_1(r), m(r)=1", "S_2(r), m(r)=1", "S_3(r), m(
plot2d: expression evaluates to non-numeric value somewhere in plo
plot2d: expression evaluates to non-numeric value somewhere in plo
plot2d: expression evaluates to non-numeric value somewhere in plo
plot2d: expression evaluates to non-numeric value somewhere in plo
```



(%t23)