

```
(%i1) kill(all);
(%o0) done
```

```
(%i1) depends([gamma, Gamma, v1], r1);
(%o1) [Gamma(r1), Gamma(r1), v1(r1)]
```

□ **1 Eq. (48)**

```
(%i2) E48: diff(gamma*m(r1), r1) = diff(1/gamma, r1);
```

```
(%o2)
```

$$m(r1) \left(\frac{d}{d r1} \Gamma \right) + \left(\frac{d}{d r1} m(r1) \right) \Gamma = - \frac{\frac{d}{d r1} \Gamma}{\Gamma^2}$$

```
(%i3) gamma: (m(r1)-v1^2/c^2)^(-1/2);
```

```
(%o3)
```

$$\frac{1}{\sqrt{m(r1) - \frac{v1^2}{c^2}}}$$

```
(%i4) ev(E48);
```

```
(%o4)
```

$$m(r1) \left(\frac{d}{d r1} \frac{1}{\sqrt{m(r1) - \frac{v1^2}{c^2}}} \right) + \frac{\frac{d}{d r1} m(r1)}{\sqrt{m(r1) - \frac{v1^2}{c^2}}} = - \left(m(r1) - \frac{v1^2}{c^2} \right) \left(\frac{d}{d r1} \frac{1}{\sqrt{m(r1) - \frac{v1^2}{c^2}}} \right)$$

```
(%i5) E48a: ev(E48, diff);
```

```
(%o5)
```

$$\frac{\frac{d}{d r1} m(r1)}{\sqrt{m(r1) - \frac{v1^2}{c^2}}} - \frac{m(r1) \left(\frac{d}{d r1} m(r1) - \frac{2 v1 \left(\frac{d}{d r1} v1 \right)}{c^2} \right)}{2 \left(m(r1) - \frac{v1^2}{c^2} \right)^{3/2}} = \frac{\frac{d}{d r1} m(r1) - \frac{2 v1 \left(\frac{d}{d r1} v1 \right)}{c^2}}{2 \sqrt{m(r1) - \frac{v1^2}{c^2}}}$$

```
re-insert gamma
```

```
(%i6) E48b: ratsubst(Gamma(r1), gamma, E48a);
```

```
(%o6)
```

$$\frac{2 \Gamma(r1)^3 m(r1) v1 \left(\frac{d}{d r1} v1 \right) + \left(2 c^2 \Gamma(r1) - c^2 \Gamma(r1)^3 m(r1) \right) \left(\frac{d}{d r1} m(r1) \right)}{2 c^2} = -$$

$$\frac{2 \Gamma(r1) v1 \left(\frac{d}{d r1} v1 \right) - c^2 \Gamma(r1) \left(\frac{d}{d r1} m(r1) \right)}{2 c^2}$$

```
(%i7) E48c: expand(E48b/Gamma(r1)*2*c^4);
```

```
(%o7)
```

$$2 c^2 \Gamma(r1)^2 m(r1) v1 \left(\frac{d}{d r1} v1 \right) - c^4 \Gamma(r1)^2 m(r1) \left(\frac{d}{d r1} m(r1) \right) + 2 c^4 \left(\frac{d}{d r1} m(r1) \right) = c^4 \left(\frac{d}{d r1} m(r1) \right) - 2 c^2 v1 \left(\frac{d}{d r1} v1 \right)$$

□ **2 Final form of (48) or (57), resp.**

```
(%i8) E48d: expand(E48c/c^4);
```

```
(%o8)
```

$$\frac{2 \Gamma(r1)^2 m(r1) v1 \left(\frac{d}{d r1} v1 \right)}{c^2} - \Gamma(r1)^2 m(r1) \left(\frac{d}{d r1} m(r1) \right) + 2 \left(\frac{d}{d r1} m(r1) \right) = \frac{d}{d r1} m(r1) - \frac{2 v1 \left(\frac{d}{d r1} v1 \right)}{c^2}$$

□ **3 General solution of (57) (transcendental eq.)**

```
(%i9) ode2(E48d, m(r1), r1);
```

```
(%o9) false
```

□ **4 Solution in plane polar coordinates**

```
(%i10) v1: sqrt(r_d^2+r1^2*phi_d^2);
```

```
(%o10)
```

$$\sqrt{r_d^2 + \varphi_d^2 r1^2}$$

```
(%i11) E48e: ev(E48d, diff);
```

```
(%o11)
```

$$-\Gamma(r1)^2 m(r1) \left(\frac{d}{d r1} m(r1) \right) + 2 \left(\frac{d}{d r1} m(r1) \right) + \frac{2 \varphi_d^2 r1 \Gamma(r1)^2 m(r1)}{c^2} = \frac{d}{d r1} m(r1) - \frac{2 \varphi_d^2 r1}{c^2}$$

```
(%i12) E48f: ode2(E48e, m(r1), r1);
(%o12) false
```

□ 5 Inertial system

```
(%i13) v1: r1_d;
(%o13) r1_d
```

```
(%i14) E48e: factor(ev(E48d, diff));
(%o14)  $-\left(\Gamma(r1)^2 m(r1) - 2\right) \left(\frac{d}{d r1} m(r1)\right) = \frac{d}{d r1} m(r1)$ 
```

```
(%i15) solve(E48e, m(r1));
(%o15)  $\left[m(r1) = \frac{1}{\Gamma(r1)^2}, \frac{d}{d r1} m(r1) = 0\right]$ 
```

```
(%i16) E48f: ode2(E48e, m(r1), r1);
(%o16)  $m(r1) = \%c$ 
```

□ 5.1 Alternatively

```
(%i17) Gamma(r1) := ev(gamma);
(%o17)  $\Gamma(r1) := \text{ev}(\Gamma)$ 
```

```
(%i18) E48e: factor(ev(E48d, diff));
(%o18)  $\frac{\left(\frac{d}{d r1} m(r1)\right) (2 r1_d^2 - c^2 m(r1))}{r1_d^2 - c^2 m(r1)} = \frac{d}{d r1} m(r1)$ 
```

```
(%i19) E48f: solve(E48e*(r1_d^2 - c^2*m(r1)), diff(m(r1), r1));
(%o19)  $\left[\frac{d}{d r1} m(r1) = 0\right]$ 
```

```
(%i20) ode2(E48e, m(r1), r1);
(%o20)  $m(r1) = \%c$ 
```

□ 5.2 Check eq. (66)

```
(%i21) E1: -1/2*Gamma(r1)^2*m(r1)+1=-1/2;
```

```
(%o21) 1 - \frac{m(r1)}{2 \left( m(r1) - \frac{r1_d^2}{c^2} \right)} = -\frac{1}{2}
```

```
(%i22) E2: expand(E1/Gamma(r1)^2);
```

```
(%o22) - \frac{m(r1)^2}{2 m(r1) - \frac{2 r1_d^2}{c^2}} + \frac{m(r1) r1_d^2}{2 c^2 m(r1) - 2 r1_d^2} - \frac{r1_d^2}{c^2} + m(r1) = \frac{r1_d^2}{2 c^2} - \frac{m(r1)}{2}
```

```
(%i23) E3: solve(E2, m(r1));
```

```
(%o23) [ m(r1) = \frac{3 r1_d^2}{2 c^2} ]
```

□ 6 Eq. (100)

```
(%i24) kill(v1);
```

```
(%o24) done
```

```
(%i25) E100: 1+m(r1)*Gamma1(r1)^2=0;
```

```
(%o25) Gamma1(r1)^2 m(r1) + 1 = 0
```

```
(%i26) Gamma1(r1) := (m(r1)-v1^2/c^2)^(-1/2);
```

```
(%o26) Gamma1(r1) := \left( m(r1) - \frac{v1^2}{c^2} \right)^{\frac{-1}{2}}
```

```
(%i27) solve(ev(E100*Gamma1(r1)^2), m(r1));
```

```
(%o27) [ m(r1) = \frac{v1^2}{2 c^2} ]
```