```
(%i1)  kill(all);
(%o0)  done
(%i1)  depends([gamma,v1],r1);
(%o1)  [Γ(r1),v1(r1)]
```

1 Eq. (48)

```
(%i2)  E48: diff(gamma*m(r1), r1) = diff(1/gamma, r1);
```

$$\text{(E48)}\quad m(r1)\left(\frac{d}{d\,r1}\,\Gamma\right)+\left(\frac{d}{d\,r1}\,m(r1)\right)\Gamma=-\frac{\frac{d}{d\,r1}\,\Gamma}{\Gamma^2}$$

```
(%i3)  gamma: (m(r1)-v1^2/c^2)^(-1/2);
```

$$\text{(gamma)}\quad \frac{1}{\sqrt{m(r1)-\frac{v1^2}{c^2}}}$$

```
(%i4)  ev(E48);
```

$$\text{(\%o4)}\quad m(r1)\left(\frac{d}{d\,r1}\,\frac{1}{\sqrt{m(r1)-\frac{v1^2}{c^2}}}\right)+\frac{\frac{d}{d\,r1}\,m(r1)}{\sqrt{m(r1)-\frac{v1^2}{c^2}}}=-\left(m(r1)-\frac{v1^2}{c^2}\right)\left(\frac{d}{d\,r1}\,\frac{1}{\sqrt{m(r1)-\frac{v1^2}{c^2}}}\right)$$

```
(%i5)  E48a: ev(E48, diff);
```

$$\text{(E48a)}\quad \frac{\frac{d}{d\,r1}\,m(r1)}{\sqrt{m(r1)-\frac{v1^2}{c^2}}}-\frac{m(r1)\left(\frac{d}{d\,r1}\,m(r1)-\frac{2\,v1\left(\frac{d}{d\,r1}\,v1\right)}{c^2}\right)}{2\left(m(r1)-\frac{v1^2}{c^2}\right)^{3/2}}=\frac{\frac{d}{d\,r1}\,m(r1)-\frac{2\,v1\left(\frac{d}{d\,r1}\,v1\right)}{c^2}}{2\sqrt{m(r1)-\frac{v1^2}{c^2}}}$$

re-insert gamma

```
(%i6)  E48b: ratsubst(%gamma, gamma, E48a);
```

$$\text{(E48b)}\quad \frac{2\,\gamma^3\,m(r1)\,v1\left(\frac{d}{d\,r1}\,v1\right)+(2\,\gamma\,c^2-\gamma^3\,c^2\,m(r1))\left(\frac{d}{d\,r1}\,m(r1)\right)}{2\,c^2}=-\frac{2\,\gamma\,v1\left(\frac{d}{d\,r1}\,v1\right)-\gamma\,c^2\left(\frac{d}{d\,r1}\,m(r1)\right)}{2\,c^2}$$

```
(%i7)  E48c: expand(E48b/'%gamma*2*c^4);
```

$$\text{(E48c)}\quad 2\,\gamma^2\,c^2\,m(r1)\,v1\left(\frac{d}{d\,r1}\,v1\right)-\gamma^2\,c^4\,m(r1)\left(\frac{d}{d\,r1}\,m(r1)\right)+2\,c^4\left(\frac{d}{d\,r1}\,m(r1)\right)=c^4\left(\frac{d}{d\,r1}\,m(r1)\right)-2\,c^2\,v1\left(\frac{d}{d\,r1}\,v1\right)$$

2 Final form of (48) or (57), resp.

```
(%i8)  E48d: expand(E48c/c^4);
```

$$\text{(E48d)}\quad \frac{2\,\gamma^2\,m(r1)\,v1\left(\frac{d}{d\,r1}\,v1\right)}{c^2}-\gamma^2\,m(r1)\left(\frac{d}{d\,r1}\,m(r1)\right)+2\left(\frac{d}{d\,r1}\,m(r1)\right)=\frac{d}{d\,r1}\,m(r1)-\frac{2\,v1\left(\frac{d}{d\,r1}\,v1\right)}{c^2}$$

3 General solution of (57) (transcendental eq.)

```
(%i9)  ode2(E48d, m(r1), r1);
```

$$\text{(\%o9)}\quad -\frac{2\,c^2\log(\gamma^2\,m(r1)+1)-\gamma^2\,c^2\,m(r1)}{2\,\gamma^2\,v1}=\int\frac{d}{d\,r1}\,v1\,d\,r1\;+\%c$$

4 Solution in plane polar coordinates

```
(%i10)  v1: sqrt(r_d^2+r1^2*phi_d^2);
```

$$\text{(v1)}\quad \sqrt{r_d^2+\varphi_d^2\,r1^2}$$

```
(%i11)  E48e: ev(E48d, diff);
```

$$\text{(E48e)}\quad -\gamma^2\,m(r1)\left(\frac{d}{d\,r1}\,m(r1)\right)+2\left(\frac{d}{d\,r1}\,m(r1)\right)+\frac{2\,\gamma^2\,\varphi_d^2\,r1\,m(r1)}{c^2}=\frac{d}{d\,r1}\,m(r1)-\frac{2\,\varphi_d^2\,r1}{c^2}$$

```
(%i12) E48f: ode2(E48e, m(r1), r1);
```

$$(E48f)\quad -\frac{2\,c^2\log(\gamma^2\,m(r1)+1)-\gamma^2\,c^2\,m(r1)}{2\,\gamma^2\,\varphi_d{}^2}=\frac{r1^2}{2}+\%c$$

```
(%i13) E48g: expand(E48f/c^2*2*phi_d^2);
```

$$(E48g)\quad m(r1)-\frac{2\log(\gamma^2\,m(r1)+1)}{\gamma^2}=\frac{\varphi_d{}^2\,r1^2}{c^2}+\frac{2\,\%c\,\varphi_d{}^2}{c^2}$$

```
(%i14) A1: factor(rhs(E48g));
```

$$(A1)\quad \frac{\varphi_d{}^2\,(r1^2+2\,\%c)}{c^2}$$

```
(%i15) E48h: lhs(E48g) = A1;
```

$$(E48h)\quad m(r1)-\frac{2\log(\gamma^2\,m(r1)+1)}{\gamma^2}=\frac{\varphi_d{}^2\,(r1^2+2\,\%c)}{c^2}$$

4.1 Replacement of %c by C

```
(%i16) E48i: expand(ratsubst(C, 2*%c/c^2, E48h));
```

$$(E48i)\quad m(r1)-\frac{2\log(\gamma^2\,m(r1)+1)}{\gamma^2}=\frac{\varphi_d{}^2\,r1^2}{c^2}+C\,\varphi_d{}^2$$

```
(%i17) E48j: lhs(E48i) = factor(rhs(E48i));
```

$$(E48j)\quad m(r1)-\frac{2\log(\gamma^2\,m(r1)+1)}{\gamma^2}=\frac{\varphi_d{}^2\,(r1^2+C\,c^2)}{c^2}$$

4.2 Determination of %c from m(r10)=m0

```
(%i18) E48j: ratsubst(m_0, m(r1), E48i);
```

$$(E48j)\quad -\frac{2\log(\gamma^2\,m_0+1)-\gamma^2\,m_0}{\gamma^2}=\frac{\varphi_d{}^2\,r1^2+C\,c^2\,\varphi_d{}^2}{c^2}$$

```
(%i19) E48k: ratsubst(r_10, r1, E48j);
```

$$(E48k)\quad -\frac{2\log(\gamma^2\,m_0+1)-\gamma^2\,m_0}{\gamma^2}=\frac{\varphi_d{}^2\,({r_{10}}^2+C\,c^2)}{c^2}$$

```
(%i20) E48l: ratsubst(phi_d0, phi_d, E48k);
```

$$(E48l)\quad -\frac{2\log(\gamma^2\,m_0+1)-\gamma^2\,m_0}{\gamma^2}=\frac{phi\_d0^2\,{r_{10}}^2+C\,c^2\,phi\_d0^2}{c^2}$$

```
(%i21) E48m: ratsubst(%gamma_0, %gamma, E48l);
```

$$(E48m)\quad -\frac{2\log(\gamma_0{}^2\,m_0+1)-\gamma_0{}^2\,m_0}{\gamma_0{}^2}=\frac{phi\_d0^2\,{r_{10}}^2+C\,c^2\,phi\_d0^2}{c^2}$$

```
(%i22) expand(solve(E48m, C));
```

$$(\%o22)\quad [\,C=-\frac{{r_{10}}^2}{c^2}-\frac{2\log(\gamma_0{}^2\,m_0+1)}{\gamma_0{}^2\,phi\_d0^2}+\frac{m_0}{phi\_d0^2}\,]$$

5 Inertial system

```
(%i23) v1: r1_d;
```

$$(v1)\quad r1_d$$

```
(%i24) E48e: ev(E48d, diff);
```

$$(E48e)\quad 2\left(\frac{d}{d\,r1}\,m(r1)\right)-\gamma^2\,m(r1)\left(\frac{d}{d\,r1}\,m(r1)\right)=\frac{d}{d\,r1}\,m(r1)$$

```
(%i25) solve(E48e, m(r1));
```

$$(\%o25)\quad [\,m(r1)=\frac{1}{\gamma^2},\frac{d}{d\,r1}\,m(r1)=0\,]$$

```
(%i26) E48f: ode2(E48e, m(r1), r1);
```

$$(E48f)\quad m(r1)=\%c$$