386(2): Simple Solution for Magnetostatics

The basic equations are:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(1)$$

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(2)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(3)$$

$$-(4)$$

The basic law of magnetostatics is:

$$\underline{B}(\underline{x}') = \frac{\mu_0}{4\pi} \int \underline{J}(\underline{x}') \times \frac{\underline{x} - \underline{x}'}{|\underline{x} - \underline{x}'|^3} d^3x'$$

It follows that:

$$\underline{B}(\underline{x}) = \frac{\mu_0}{4\pi} \underline{\nabla} \times \int \frac{\underline{J}(\underline{x}')}{|\underline{x} - \underline{x}'|} d^3x' \quad -(5)$$

From eqs. (1) and (5):

$$\underline{\omega} \times \underline{A} = \underline{\nabla} \times \left(\underline{A} - \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{x}')}{|\underline{x} - \underline{x}'|} d^3x' \right) \quad -(6)$$

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{A} = \underline{0} \quad -(7)$$

Eqs. (6) and (7) must conserve antisymmetry ~~(6)~~, Eq. (7) follows directly from eq. (6), and in general, from the antisymmetry equations:

$$\underline{\omega} \times \underline{A} \neq \underline{0} \quad -(7)$$

It follows at the ECE2 level that:

$$\underline{A} \neq \frac{\mu_0}{4\pi} \int \frac{\underline{J}(x')}{|\underline{x} - \underline{x}'|} d^3x' \quad -(8)$$

2) So the usual standard model:

$$\underline{B} = \underline{\nabla} \times \underline{\alpha} \quad -(9)$$

and

$$\underline{\alpha} = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(x')}{|\underline{x} - \underline{x}'|} d^3x' \quad -(10)$$

cannot be used.

The methodology should be the same as in immediately preceding ECE2 papers and start with the experimentally observed $\underline{B}$. So:

$$\underline{B}(\text{exp}) = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(11)$$

and

$$\underline{\nabla} \cdot \underline{B}(\text{exp}) = -\underline{\nabla} \cdot \underline{\omega} \times \underline{A} = 0 \quad -(12)$$

therefore

$$\boxed{\underline{\omega} \times \underline{A} = \underline{\nabla} \times \underline{A} - \underline{B}(\text{exp})} \quad -(13)$$

The left hand side of eq. (13) is evaluated from the antisymmetry equations:

$$\left(\frac{\partial}{\partial Y} - \omega_Y \right) A_Z = -\left(\frac{\partial}{\partial Z} - \omega_Z \right) A_Y \quad -(14)$$

$$\left(\frac{\partial}{\partial Z} - \omega_Z \right) A_X = -\left(\frac{\partial}{\partial X} - \omega_X \right) A_Z \quad -(15)$$

$$\left(\frac{\partial}{\partial X} - \omega_X \right) A_Y = -\left(\frac{\partial}{\partial Y} - \omega_Y \right) A_X \quad -(16)$$

Given $\underline{A}$, the spin connection $\underline{\omega}$ can be found from eqs. (14) to (16).

Therefore $\underline{\omega}$ and $\underline{A}$ can always be chosen so that (scp) is produced from eq (11). In general, eqs. (11) and (14) to (16) must be solved simultaneously for $\underline{\omega}$ and $\underline{A}$. This is possible using the computer.

— (17)

Examples

1) Standard model

$$\underline{\omega} = \underline{0} \; ; \; \underline{A} = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{x}')}{|\underline{x} - \underline{x}'|} d^3 x'$$

2) Static Magnetic Field

$$\underline{\omega} = -\left(\frac{1}{X} \underline{i} + \frac{1}{Y} \underline{j} \right), \quad -(18)$$

$$\underline{A} = \frac{B^{(0)}}{2} \left(-Y \underline{i} + X \underline{j} \right) \quad -(19)$$

$$\underline{B} = 2 B^{(0)} \underline{k} \quad -(20)$$

3) Assume for simplicity that:

$$\underline{\nabla} \times \underline{A} = -\underline{\omega} \times \underline{A} \quad -(21)$$

then

$$\underline{B} = 2 \underline{\nabla} \times \underline{A} \quad -(22)$$

$$= \underline{\nabla} \times \underline{\alpha}$$

so

$$\underline{A} = \frac{1}{2} \underline{\alpha} \quad -(23)$$

and

$$\underline{B} = -2 \underline{\omega} \times \underline{A} = -\underline{\omega} \times \underline{\alpha} \quad -(24)$$

$$= \underline{\nabla} \times \underline{\alpha}$$

Therefore various models for $\underline{\alpha}$ can

4) be used and $\underline{\omega}$ worked out from the antisymmetry relations (14) to (16) applied to $\underline{\alpha}$.

In general:

$$\alpha_\phi(r,\theta) = \frac{\mu_0 I a}{4\pi}\int_0^{2\pi} \frac{\cos\phi' \, d\phi'}{(a^2+r^2-2ar\sin\theta\cos\phi')^{1/2}} \quad -(17)$$

and

$$\underline{\omega} \times \underline{A} = \frac{1}{2}\,\underline{\omega}\times\underline{\alpha} \quad -(18)$$

can be worked out.
