1) Plane Wave Solution for the Spin Connection

In the case of electromagnetism:

$$\nabla \cdot \mathbf{B} = 0 \qquad - (1)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \qquad - (2)$$

$$\mathbf{B} = \nabla \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} \qquad - (3)$$

$$\mathbf{E} = - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} \qquad - (4)$$

and by antisymmetry:

$$\left( \frac{\partial}{\partial Y} - \omega_Y \right) A_Z = - \left( \frac{\partial}{\partial Z} - \omega_Z \right) A_Y \qquad - (5)$$

$$\left( \frac{\partial}{\partial Z} - \omega_Z \right) A_X = - \left( \frac{\partial}{\partial X} - \omega_X \right) A_Z \qquad - (6)$$

$$\left( \frac{\partial}{\partial X} - \omega_X \right) A_Y = - \left( \frac{\partial}{\partial Y} - \omega_Y \right) A_X \qquad - (7)$$

From eqs. (2) to (4):

$$\frac{\partial}{\partial t} \left( \boldsymbol{\omega} \times \mathbf{A} \right) + \nabla \times \left( \omega_0 \mathbf{A} \right) = \mathbf{0} \qquad - (8)$$

Consider the plane wave:

$$\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left( \mathbf{i} - i \mathbf{j} \right) \exp\left( -i(\omega t - \kappa Z) \right) \qquad - (9)$$

The antisymmetry equations (5) to (7) reduce to a simpler set of equations:

2)

$$\frac{\partial A_X^{(1)}}{\partial Z} = \omega_Z A_X^{(1)} \quad -(10)$$

$$\frac{\partial A_Y^{(1)}}{\partial Z} = \omega_Z A_Y^{(1)} \quad -(11)$$

$$-\omega_X A_Y^{(1)} = \omega_Y A_X^{(1)} \quad -(12)$$

where

$$A_X^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \exp\left(-i(\omega t - \kappa Z)\right) \quad -(13)$$

$$A_Y^{(1)} = -i\frac{A^{(0)}}{\sqrt{2}} \exp\left(-i(\omega t - \kappa Z)\right) \quad -(14)$$

$$A_Z^{(1)} = 0 \quad -(15).$$

Fm eqs. (10) and (13):

$$\omega_Z = i\kappa. \quad -(16)$$

Fm eqs. (11) and (14):

$$\omega_Z = \kappa \quad -(17)$$

So

$$\omega_Z = 0 \quad -(18)$$

is the only possibility.

Fm eqs. (12) to (14):

$$i\omega_X = \omega_Y \quad -(19)$$

Eqs. (18) and (19) are satisfied by:

$$\boxed{\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \exp\left(i(\omega t - \kappa Z)\right)} \quad -(20)$$

) Therefore the space part of the spin connection four vector

a plane wave.

In order to find $\omega_0$, it is necessary to solve:

$$\frac{\partial}{\partial t}\left(\underline{\omega}^{(2)}\times\underline{A}^{(1)}\right)+\underline{\nabla}\times\left(\omega_0\underline{A}^{(1)}\right)=\underline{0}\quad -(21)$$

Here:

$$\underline{\omega}^{(2)}\times\underline{A}^{(1)}=\frac{\omega^{(0)}A^{(0)}}{2}\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 1 & i & 0\\ 1 & -i & 0\end{vmatrix}\quad -(22)$$

$$=-i\omega^{(0)}A^{(0)}\underline{k}$$

So:

$$\frac{\partial}{\partial t}\left(\underline{\omega}^{(2)}\times\underline{A}^{(1)}\right)=\underline{0}\quad -(23)$$

and

$$\underline{\nabla}\times\left(\omega_0\underline{A}^{(1)}\right)=\underline{0}\quad -(24)$$

This means that:

$$\omega_0\underline{\nabla}\times\underline{A}^{(1)}+\left(\underline{\nabla}\omega_0\right)\times\underline{A}^{(1)}=\underline{0}\quad -(25)$$

in which

$$\underline{\nabla}\times\underline{A}^{(1)}=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x^{(1)} & A_y^{(1)} & 0\end{vmatrix}\quad -(26)$$

So

$$\omega_0 \underline{\nabla} \times \underline{A}^{(1)} = -\omega_0 \frac{\partial A_Y^{(1)}}{\partial Z} \underline{i} + \omega_0 \frac{\partial A_X^{(1)}}{\partial Z} \underline{j} \quad -(27)$$

Assume that

$$\omega_0 = \omega^{(0)} \exp\left(-i(\omega t - \kappa Z)\right) \quad -(28)$$

then

$$\underline{\nabla} \omega_0 = \frac{\partial \omega_0}{\partial Z} \underline{k} \quad -(29)$$

and

$$\underline{\nabla} \omega_0 \times \underline{A}^{(1)} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & \frac{\partial \omega_0}{\partial Z} \\ A_X^{(1)} & A_Y^{(1)} & 0 \end{vmatrix} \quad -(29)$$

$$= -\frac{\partial \omega_0}{\partial Z} A_Y^{(1)} \underline{i} - \frac{\partial \omega_0}{\partial Z} A_X^{(1)} \underline{j} \quad -(30)$$

Fm eqs (27) and (30):

$$\omega_0 \frac{\partial A_Y^{(1)}}{\partial Z} = \frac{\partial \omega_0}{\partial Z} A_Y^{(1)} \quad -(31)$$

$$\omega_0 \frac{\partial A_X^{(1)}}{\partial Z} = -\frac{\partial \omega_0}{\partial Z} A_X^{(1)} \quad -(32)$$

We have:

$$\frac{\partial A_X^{(1)}}{\partial Z} = i\kappa A_X^{(1)} ; \quad \frac{\partial A_Y^{(1)}}{\partial Z} = i\kappa A_Y^{(1)} \quad -(33)$$

so

$$\frac{\partial \omega_0}{\partial Z} = i\kappa \quad -(34)$$

and

$$\frac{\partial \omega_0}{\partial Z} = -i\kappa \quad -(35)$$

i.e

$$\omega_0 = \omega^{(0)} = 0 \quad -(36)$$

5) Summary

If:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)\exp\left(-i(\omega t - \kappa Z)\right) \quad - (37)$$

then a possible solution of eqs. (1) to (4) is:

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)\exp\left(i(\omega t - \kappa Z)\right) \quad - (38)$$

so

$$i\omega_X = \omega_Y \quad - (39)$$

$$\omega_Z = 0 \quad - (40)$$

So the space part of the spin connection is a plane wave. The time part of the spin connection is:

$$\omega_0 = 0 \quad - (41)$$

The electric field strength $\underline{E}^{(1)}$ and the magnetic flux density $\underline{B}^{(1)}$ are also plane waves.

The spin connection plane wave is a wave of aether or spacetime. Eqs. (1) to (4) are the homogeneous field equations of the spacetime or aether. The plane wave (38) is the same for gravitation and electromagnetism.