

```
(%i1) kill(all);
(%o0) done

(%i1) cross(a,b) := [a[2]*b[3] - a[3]*b[2],
                    a[3]*b[1] - a[1]*b[3],
                    a[1]*b[2] - a[2]*b[1]];
(%o1) cross(a,b) := [a2 b3 - a3 b2, a3 b1 - a1 b3, a1 b2 - a2 b1]

(%i2) curl(a) := [diff(a[3],y) - diff(a[2],z),
                  diff(a[1],z) - diff(a[3],x),
                  diff(a[2],x) - diff(a[1],y)];
(%o2) curl(a) := [ d/dy a3 - d/dz a2, d/dz a1 - d/dx a3, d/dx a2 - d/dy a1]

(%i3) grad(psi) := [diff(psi,x), diff(psi,y), diff(psi,z)];
(%o3) grad(Psi) := [ d/dx Psi, d/dy Psi, d/dz Psi]
```

□ 1 Equation set of 7 equations

```
(%i4) depends([omega_0, omega_x, omega_y, omega_z, Q_x, Q_y, Q_z, Phi], [x, y, z, t])
(%o4) [w0(x, y, z, t), wx(x, y, z, t), wy(x, y, z, t), wz(x, y, z, t),
       Qx(x, y, z, t), Qy(x, y, z, t), Qz(x, y, z, t), Phi(x, y, z, t)]

(%i6) omega: [omega_x, omega_y, omega_z];
       Q: [Q_x, Q_y, Q_z];
(%o5) [wx, wy, wz]
(%o6) [Qx, Qy, Qz]
```

□ 1.1 Eq. (15)

```
(%i7) E15: diff(cross(omega,Q),t) = -omega_0*curl(Q) - cross(grad(omega_0),Q)
(%o7) [-Qy (d/dt wz) - (d/dt Qy) wz + Qz (d/dt wy) + (d/dt Qz) wy, Qx
       (d/dt wz) + (d/dt Qx) wz - Qz (d/dt wx) - (d/dt Qz) wx, -Qx (d/dt wy) - (d/dt Qx)
       wy + Qy (d/dt wx) + (d/dt Qy) wx] = [Qy (d/dz w0) - Qz (d/dy w0) -
       (d/dy Qz - d/dz Qy) w0, -Qx (d/dz w0) + Qz (d/dx w0) - (d/dz Qx - d/dx Qz) w0, Qx
       (d/dy w0) - Qy (d/dx w0) - (d/dx Qy - d/dy Qx) w0]
```

□ 1.2 3 Eqs. from Eq. (15)

```
(%i10) E15a: first(lhs(E15))=first(rhs(E15));
E15b: second(lhs(E15))=second(rhs(E15));
E15c: third(lhs(E15))=third(rhs(E15));
```

$$\begin{aligned}
(\%o8) \quad & -Q_y \left(\frac{d}{dt} \omega_z \right) - \left(\frac{d}{dt} Q_y \right) \omega_z + Q_z \left(\frac{d}{dt} \omega_y \right) + \left(\frac{d}{dt} Q_z \right) \omega_y = Q_y \left(\frac{d}{dz} \omega_0 \right) \\
& - Q_z \left(\frac{d}{dy} \omega_0 \right) - \left(\frac{d}{dy} Q_z - \frac{d}{dz} Q_y \right) \omega_0 \\
(\%o9) \quad & Q_x \left(\frac{d}{dt} \omega_z \right) + \left(\frac{d}{dt} Q_x \right) \omega_z - Q_z \left(\frac{d}{dt} \omega_x \right) - \left(\frac{d}{dt} Q_z \right) \omega_x = -Q_x \left(\frac{d}{dz} \omega_0 \right) \\
& + Q_z \left(\frac{d}{dx} \omega_0 \right) - \left(\frac{d}{dz} Q_x - \frac{d}{dx} Q_z \right) \omega_0 \\
(\%o10) \quad & -Q_x \left(\frac{d}{dt} \omega_y \right) - \left(\frac{d}{dt} Q_x \right) \omega_y + Q_y \left(\frac{d}{dt} \omega_x \right) + \left(\frac{d}{dt} Q_y \right) \omega_x = Q_x \left(\frac{d}{dy} \omega_0 \right) \\
& - Q_y \left(\frac{d}{dx} \omega_0 \right) - \left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x \right) \omega_0
\end{aligned}$$

□ **1.3 Eqs. (19–22)**

```
(%i13) E19: diff(Q_z,y) - omega_y*Q_z = -(diff(Q_y,z) - omega_z*Q_y);
E20: diff(Q_x,z) - omega_z*Q_x = -(diff(Q_z,x) - omega_x*Q_z);
E21: diff(Q_y,x) - omega_x*Q_y = -(diff(Q_x,y) - omega_y*Q_x);
```

$$\begin{aligned}
(\%o11) \quad & \frac{d}{dy} Q_z - Q_z \omega_y = Q_y \omega_z - \frac{d}{dz} Q_y \\
(\%o12) \quad & \frac{d}{dz} Q_x - Q_x \omega_z = Q_z \omega_x - \frac{d}{dx} Q_z \\
(\%o13) \quad & \frac{d}{dx} Q_y - Q_y \omega_x = Q_x \omega_y - \frac{d}{dy} Q_x
\end{aligned}$$

□ **1.4 Eq. (23)**

```
(%i14) E23: Q_x*(diff(omega_z,y)-diff(omega_y,z))+Q_y*(diff(omega_x,z)-diff(omega_z,x))
+Q_z*(diff(omega_y,x)-diff(omega_x,y))
= omega_x*(diff(Q_z,y)-diff(Q_y,z))+omega_y*(diff(Q_x,z)-diff(Q_z,x))
+omega_z*(diff(Q_y,x)-diff(Q_x,y));
```

$$\begin{aligned}
(\%o14) \quad & Q_x \left(\frac{d}{dy} \omega_z - \frac{d}{dz} \omega_y \right) + Q_y \left(\frac{d}{dz} \omega_x - \frac{d}{dx} \omega_z \right) + Q_z \left(\frac{d}{dx} \omega_y - \frac{d}{dy} \omega_x \right) = \\
& \left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x \right) \omega_z + \left(\frac{d}{dz} Q_x - \frac{d}{dx} Q_z \right) \omega_y + \left(\frac{d}{dy} Q_z - \frac{d}{dz} Q_y \right) \omega_x
\end{aligned}$$

```
(%i15) feval(Q_x, Q_y, Q_z, omega_x, omega_y, omega_z, omega_0) := (
    print("E15a: ", R1: ev(E15a, diff)),
    print("E15b: ", R2: ev(E15b, diff)),
    print("E15c: ", R3: ev(E15c, diff)),
    print("E19: ", R4: ev(E19, diff)),
    print("E20: ", R5: ev(E20, diff)),
    print("E21: ", R6: ev(E21, diff)),
    print("E23: ", R7: ev(E23, diff)), ""
);
```

```
(%o15) feval(Q_x, Q_y, Q_z, omega_x, omega_y, omega_z, omega_0) := (
print(E15a: , R1: ev(E15a, diff)), print(E15b: , R2: ev(E15b, diff)),
print(E15c: , R3: ev(E15c, diff)), print(E19: , R4: ev(E19, diff)),
print(E20: , R5: ev(E20, diff)), print(E21: , R6: ev(E21, diff)),
print(E23: , R7: ev(E23, diff)), )
```

□ 2 Special cases

□ 2.1 Q (with 3 oscillatory components)

```
(%i16) kill(omega_0, omega_x, omega_y, omega_z, Q_x, Q_y, Q_z, Phi, Q_0);
(%o16) done
```

```
(%i17) depends([omega_0], [t], [omega_x, omega_y, omega_z, Q_x, Q_y, Q_z, Phi], [x,
(%o17) [omega_0(t), omega_x(x, y, z), omega_y(x, y, z), omega_z(x, y, z), Q_x(x, y, z), Q_y(x, y, z),
Q_z(x, y, z), Phi(x, y, z)]
```

```
(%i20) Q_x: Q_1*cos(%beta*t-(k_x*x+k_y*y+k_z*z));
Q_y: Q_2*cos(%beta*t-(k_x*x+k_y*y+k_z*z));
Q_z: Q_3*cos(%beta*t-(k_x*x+k_y*y+k_z*z));
```

```
(%o18) Q_1 cos(k_z z + k_y y + k_x x - beta t)
(%o19) Q_2 cos(k_z z + k_y y + k_x x - beta t)
(%o20) Q_3 cos(k_z z + k_y y + k_x x - beta t)
```

```
(%i21) feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0);
E15a:  beta Q3 omega_y sin(k_z z+k_y y+k_x x-beta t)-beta Q2 omega_z sin(k_z z+k_y y+k_x x-beta t)=-
omega_0 (Q2 k_z sin(k_z z+k_y y+k_x x-beta t)-Q3 k_y sin(k_z z+k_y y+k_x x-beta t))
E15b:  beta Q1 omega_z sin(k_z z+k_y y+k_x x-beta t)-beta Q3 omega_x sin(k_z z+k_y y+k_x x-beta t)=-
omega_0 (Q3 k_x sin(k_z z+k_y y+k_x x-beta t)-Q1 k_z sin(k_z z+k_y y+k_x x-beta t))
E15c:  beta Q2 omega_x sin(k_z z+k_y y+k_x x-beta t)-beta Q1 omega_y sin(k_z z+k_y y+k_x x-beta t)=-
omega_0 (Q1 k_y sin(k_z z+k_y y+k_x x-beta t)-Q2 k_x sin(k_z z+k_y y+k_x x-beta t))
E19:   -Q3 k_y sin(k_z z+k_y y+k_x x-beta t)-Q3 omega_y cos(k_z z+k_y y+k_x x-beta t)=Q2 k_z
sin(k_z z+k_y y+k_x x-beta t)+Q2 omega_z cos(k_z z+k_y y+k_x x-beta t)
E20:   -Q1 k_z sin(k_z z+k_y y+k_x x-beta t)-Q1 omega_z cos(k_z z+k_y y+k_x x-beta t)=Q3 k_x
sin(k_z z+k_y y+k_x x-beta t)+Q3 omega_x cos(k_z z+k_y y+k_x x-beta t)
E21:   -Q2 k_x sin(k_z z+k_y y+k_x x-beta t)-Q2 omega_x cos(k_z z+k_y y+k_x x-beta t)=Q1 k_y
sin(k_z z+k_y y+k_x x-beta t)+Q1 omega_y cos(k_z z+k_y y+k_x x-beta t)
E23:   Q1 (d/dy omega_z - d/dz omega_y) cos(k_z z+k_y y+k_x x-beta t)+Q2 (d/dz omega_x - d/dx omega_z)
cos(k_z z+k_y y+k_x x-beta t)+Q3 (d/dx omega_y - d/dy omega_x) cos(k_z z+k_y y+k_x x-beta t)=omega_x
(Q2 k_z sin(k_z z+k_y y+k_x x-beta t)-Q3 k_y sin(k_z z+k_y y+k_x x-beta t))+omega_y
(Q3 k_x sin(k_z z+k_y y+k_x x-beta t)-Q1 k_z sin(k_z z+k_y y+k_x x-beta t))+omega_z
(Q1 k_y sin(k_z z+k_y y+k_x x-beta t)-Q2 k_x sin(k_z z+k_y y+k_x x-beta t))
(%o21)
```

```
(%i28) ratsimp(R1/sin(k_z*z+k_y*y+k_x*x-%beta*t));
ratsimp(R2/sin(k_z*z+k_y*y+k_x*x-%beta*t));
ratsimp(R3/sin(k_z*z+k_y*y+k_x*x-%beta*t));
ratsimp(R4);
ratsimp(R5);
ratsimp(R6);
ratsimp(R7);

(%o22)  $\beta Q_3 \omega_y - \beta Q_2 \omega_z = (Q_3 k_y - Q_2 k_z) \omega_0$ 
(%o23)  $\beta Q_1 \omega_z - \beta Q_3 \omega_x = (Q_1 k_z - Q_3 k_x) \omega_0$ 
(%o24)  $\beta Q_2 \omega_x - \beta Q_1 \omega_y = (Q_2 k_x - Q_1 k_y) \omega_0$ 
(%o25)  $-Q_3 k_y \sin(k_z z + k_y y + k_x x - \beta t) - Q_3 \omega_y \cos(k_z z + k_y y + k_x x - \beta t) = Q_2$ 
 $k_z \sin(k_z z + k_y y + k_x x - \beta t) + Q_2 \omega_z \cos(k_z z + k_y y + k_x x - \beta t)$ 
(%o26)  $-Q_1 k_z \sin(k_z z + k_y y + k_x x - \beta t) - Q_1 \omega_z \cos(k_z z + k_y y + k_x x - \beta t) = Q_3$ 
 $k_x \sin(k_z z + k_y y + k_x x - \beta t) + Q_3 \omega_x \cos(k_z z + k_y y + k_x x - \beta t)$ 
(%o27)  $-Q_2 k_x \sin(k_z z + k_y y + k_x x - \beta t) - Q_2 \omega_x \cos(k_z z + k_y y + k_x x - \beta t) = Q_1$ 
 $k_y \sin(k_z z + k_y y + k_x x - \beta t) + Q_1 \omega_y \cos(k_z z + k_y y + k_x x - \beta t)$ 
(%o28)  $(Q_1 \left(\frac{d}{d y} \omega_z\right) - Q_2 \left(\frac{d}{d x} \omega_z\right) - Q_1 \left(\frac{d}{d z} \omega_y\right) + Q_3 \left(\frac{d}{d x} \omega_y\right) + Q_2 \left(\frac{d}{d z} \omega_x\right) - Q_3 \left(\frac{d}{d y} \omega_x\right)) \cos(k_z z + k_y y + k_x x - \beta t) =$ 
 $((Q_1 k_y - Q_2 k_x) \omega_z + (Q_3 k_x - Q_1 k_z) \omega_y + (Q_2 k_z - Q_3 k_y) \omega_x)$ 
 $\sin(k_z z + k_y y + k_x x - \beta t)$ 
```

□ 2.2 Q (with 2 osc. components), omega=const.

```
(%i29) kill(omega_0, omega_x, omega_y, omega_z, Q_x, Q_y, Q_z, Phi, Q_0);
(%o29) done
```

```
(%i30) depends([omega_0], [t], [omega_x, omega_y, omega_z, Q_x, Q_y, Q_z, Phi], [x,
(%o30) [ $\omega_0(t)$ ,  $\omega_x(x, y, z)$ ,  $\omega_y(x, y, z)$ ,  $\omega_z(x, y, z)$ ,  $Q_x(x, y, z)$ ,  $Q_y(x, y, z)$ ,
 $Q_z(x, y, z)$ ,  $\Phi(x, y, z)$ ]
```

```
(%i33) Q_x: Q_1*cos(%beta*t-(k_x*x+k_y*y));
Q_y: Q_2*sin(%beta*t-(k_x*x+k_y*y));
Q_z: Q_3;

(%o31)  $Q_1 \cos(k_y y + k_x x - \beta t)$ 
(%o32)  $-Q_2 \sin(k_y y + k_x x - \beta t)$ 
(%o33)  $Q_3$ 
```

```
(%i36) omega_x: 0;
omega_y: 0;
omega_z: kappa;

(%o34) 0
(%o35) 0
(%o36)  $\kappa$ 
```

```
(%i37) feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0);
E15a: -beta Q2 kappa cos(k_y y+k_x x-beta t)=0
E15b: beta Q1 kappa sin(k_y y+k_x x-beta t)=0
E15c: 0=-omega_0 (Q1 k_y sin(k_y y+k_x x-beta t)-Q2 k_x cos(k_y y+k_x x-beta t))
E19: 0=-Q2 kappa sin(k_y y+k_x x-beta t)
E20: -Q1 kappa cos(k_y y+k_x x-beta t)=0
E21: -Q2 k_x cos(k_y y+k_x x-beta t)=Q1 k_y sin(k_y y+k_x x-beta t)
E23: 0=kappa (Q1 k_y sin(k_y y+k_x x-beta t)-Q2 k_x cos(k_y y+k_x x-beta t))
(%o37)
```

```
(%i44) E1: ratsimp(R1/cos(k_y*y+k_x*x-%beta*t));
E2: ratsimp(R2/sin(k_y*y+k_x*x-%beta*t));
E3: ratsimp(R3/sin(k_y*y+k_x*x-%beta*t));
E4: ratsimp(R4/sin(k_y*y+k_x*x-%beta*t));
E5: ratsimp(R5/cos(k_y*y+k_x*x-%beta*t));
E6: ratsimp(R6/sin(k_y*y+k_x*x-%beta*t));
E7: ratsimp(R7/sin(k_y*y+k_x*x-%beta*t));
(%o38) -beta Q2 kappa=0
(%o39) beta Q1 kappa=0
(%o40) 0=-frac(Q1 k_y omega_0 sin(k_y y+k_x x-beta t)-Q2 k_x omega_0 cos(k_y y+k_x x-beta t))
sin(k_y y+k_x x-beta t)
(%o41) 0=-Q2 kappa
(%o42) -Q1 kappa=0
(%o43) -frac(Q2 k_x cos(k_y y+k_x x-beta t))
sin(k_y y+k_x x-beta t) = Q1 k_y
(%o44) 0=frac(Q1 k_y kappa sin(k_y y+k_x x-beta t)-Q2 k_x kappa cos(k_y y+k_x x-beta t))
sin(k_y y+k_x x-beta t)
```

```
(%i45) solve([E3,E2,E1], [Q1,Q2,Q3]);
(%o45) []
```

```
(%i46) solve([E1,E2,E7], [k_x,k_y,k_z]);
(%o46) []
```

□ 2.3 general Q, omega=const.

```
(%i47) kill(omega_0,omega_x,omega_y,omega_z,Q_x,Q_y,Q_z,Phi,Q_0);
(%o47) done
```

```
(%i48) depends([omega_0],[t],[omega_x,omega_y,omega_z,Q_x,Q_y,Q_z,Phi], [x,
(%o48) [omega_0(t), omega_x(x,y,z,t), omega_y(x,y,z,t), omega_z(x,y,z,t), Q_x(x,y,z,t),
Q_y(x,y,z,t), Q_z(x,y,z,t), Phi(x,y,z,t)]
```

```
(%i51)  omega_x: 0;
        omega_y: 0;
        omega_z: kappa;
(%o49)  0
(%o50)  0
(%o51)   $\kappa$ 

(%i52)  feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0);
E15a:   $-\left(\frac{d}{dt} Q_y\right) \kappa = -\left(\frac{d}{dy} Q_z - \frac{d}{dz} Q_y\right) \omega_0$ 
E15b:   $\left(\frac{d}{dt} Q_x\right) \kappa = -\left(\frac{d}{dz} Q_x - \frac{d}{dx} Q_z\right) \omega_0$ 
E15c:   $0 = -\left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x\right) \omega_0$ 
E19:    $\frac{d}{dy} Q_z = Q_y \kappa - \frac{d}{dz} Q_y$ 
E20:    $\frac{d}{dz} Q_x - Q_x \kappa = -\frac{d}{dx} Q_z$ 
E21:    $\frac{d}{dx} Q_y = -\frac{d}{dy} Q_x$ 
E23:    $0 = \left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x\right) \kappa$ 
(%o52)
```

□ 2.4 Additionally: $Q_x = Q_y = f(t)$ only

```
(%i55)  Q_x: q*sin(beta*t);
        Q_y: q*cos(beta*t);
        Q_z: a(t)*x+b(t)*y;
(%o53)  q sin(beta t)
(%o54)  q cos(beta t)
(%o55)  b(t) y+a(t) x

(%i56)  feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0);
E15a:   $\beta \kappa q \sin(\beta t) = -\omega_0 b(t)$ 
E15b:   $\beta \kappa q \cos(\beta t) = \omega_0 a(t)$ 
E15c:  0=0
E19:    $b(t) = \kappa q \cos(\beta t)$ 
E20:    $-\kappa q \sin(\beta t) = -a(t)$ 
E21:   0=0
E23:   0=0
(%o56)

(%i57)  Omega: curl([Q_x,Q_y,Q_z])-cross([0,0,kappa],[Q_x,Q_y,Q_z]);
(%o57)  [ $\kappa q \cos(\beta t) + b(t)$ ,  $-\kappa q \sin(\beta t) - a(t)$ , 0]

(%i58)  ev(Omega, [a(t)=kappa*q*cos(beta*t), b(t)=-kappa*q*sin(beta*t)]);
(%o58)  [ $\kappa q \cos(\beta t) - \kappa q \sin(\beta t)$ ,  $-\kappa q \sin(\beta t) - \kappa q \cos(\beta t)$ , 0]
```

```
(%i59) Omega: (-sqrt(2)*kappa*q*[sin(%beta*t-%pi/4), sin(%beta*t+%pi/4), 0]
(%o59) [-sqrt(2) kappa q sin(beta t - pi/4), -sqrt(2) kappa q sin(beta t + pi/4), 0]
```