

```

(%i1) kill(all);
(%o0) done

(%i1) cross(a,b) := [a[2]*b[3] - a[3]*b[2],
                    a[3]*b[1] - a[1]*b[3],
                    a[1]*b[2] - a[2]*b[1]];
(%o1) cross(a,b) := [a2 b3 - a3 b2, a3 b1 - a1 b3, a1 b2 - a2 b1]

(%i2) curl(a) := [diff(a[3],y) - diff(a[2],z),
                  diff(a[1],z) - diff(a[3],x),
                  diff(a[2],x) - diff(a[1],y)];
(%o2) curl(a) := [d/dy a3 - d/dz a2, d/dz a1 - d/dx a3, d/dx a2 - d/dy a1]

(%i3) grad(psi) := [diff(psi,x), diff(psi,y), diff(psi,z)];
(%o3) grad(Psi) := [d/dx Psi, d/dy Psi, d/dz Psi]

```

□ 1 Equation set of 7 equations

```

(%i4) depends([omega_0, omega_x, omega_y, omega_z, Q_x, Q_y, Q_z, Phi], [x,y,z,t]);
(%o4) [w0(x,y,z,t), wx(x,y,z,t), wy(x,y,z,t), wz(x,y,z,t),
Qx(x,y,z,t), Qy(x,y,z,t), Qz(x,y,z,t), Phi(x,y,z,t)]

(%i6) omega: [omega_x, omega_y, omega_z];
      Q: [Q_x, Q_y, Q_z];
(%o5) [wx, wy, wz]
(%o6) [Qx, Qy, Qz]

```

□ 1.1 Eq. (15)

```

(%i7) E15: diff(cross(omega,Q),t) = -omega_0*curl(Q)-cross(grad(omega_0),Q);
(%o7) [-Qy (d/dt wz) - (d/dt Qy) wz + Qz (d/dt wy) + (d/dt Qz) wy, Qx
(d/dt wz) + (d/dt Qx) wz - Qz (d/dt wx) - (d/dt Qz) wx, -Qx (d/dt wy) - (d/dt Qx)
wy + Qy (d/dt wx) + (d/dt Qy) wx] = [Qy (d/dz w0) - Qz (d/dy w0) -
(d/dy Qz - d/dz Qy) w0, -Qx (d/dz w0) + Qz (d/dx w0) - (d/dz Qx - d/dx Qz) w0, Qx
(d/dy w0) - Qy (d/dx w0) - (d/dx Qy - d/dy Qx) w0]

```

□ 1.2 3 Eqs. from Eq. (15)

```
(%i10) E15a: first(lhs(E15))=first(rhs(E15));
E15b: second(lhs(E15))=second(rhs(E15));
E15c: third(lhs(E15))=third(rhs(E15));
```

$$\begin{aligned}
(\%o8) \quad & -Q_y \left(\frac{d}{dt} \omega_z \right) - \left(\frac{d}{dt} Q_y \right) \omega_z + Q_z \left(\frac{d}{dt} \omega_y \right) + \left(\frac{d}{dt} Q_z \right) \omega_y = Q_y \left(\frac{d}{dz} \omega_0 \right) \\
& - Q_z \left(\frac{d}{dy} \omega_0 \right) - \left(\frac{d}{dy} Q_z - \frac{d}{dz} Q_y \right) \omega_0 \\
(\%o9) \quad & Q_x \left(\frac{d}{dt} \omega_z \right) + \left(\frac{d}{dt} Q_x \right) \omega_z - Q_z \left(\frac{d}{dt} \omega_x \right) - \left(\frac{d}{dt} Q_z \right) \omega_x = -Q_x \left(\frac{d}{dz} \omega_0 \right) \\
& + Q_z \left(\frac{d}{dx} \omega_0 \right) - \left(\frac{d}{dz} Q_x - \frac{d}{dx} Q_z \right) \omega_0 \\
(\%o10) \quad & -Q_x \left(\frac{d}{dt} \omega_y \right) - \left(\frac{d}{dt} Q_x \right) \omega_y + Q_y \left(\frac{d}{dt} \omega_x \right) + \left(\frac{d}{dt} Q_y \right) \omega_x = Q_x \left(\frac{d}{dy} \omega_0 \right) \\
& - Q_y \left(\frac{d}{dx} \omega_0 \right) - \left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x \right) \omega_0
\end{aligned}$$

1.3 Eqs. (19-22)

```
(%i13) E19: diff(Q_z,y) - omega_y*Q_z = -(diff(Q_y,z) - omega_z*Q_y);
E20: diff(Q_x,z) - omega_z*Q_x = -(diff(Q_z,x) - omega_x*Q_z);
E21: diff(Q_y,x) - omega_x*Q_y = -(diff(Q_x,y) - omega_y*Q_x);
```

$$\begin{aligned}
(\%o11) \quad & \frac{d}{dy} Q_z - Q_z \omega_y = Q_y \omega_z - \frac{d}{dz} Q_y \\
(\%o12) \quad & \frac{d}{dz} Q_x - Q_x \omega_z = Q_z \omega_x - \frac{d}{dx} Q_z \\
(\%o13) \quad & \frac{d}{dx} Q_y - Q_y \omega_x = Q_x \omega_y - \frac{d}{dy} Q_x
\end{aligned}$$

1.4 Eq. (23)

```
(%i14) E23: Q_x*(diff(omega_z,y)-diff(omega_y,z))+Q_y*(diff(omega_x,z)-diff(omega_z,x))
+Q_z*(diff(omega_y,x)-diff(omega_x,y))
= omega_x*(diff(Q_z,y)-diff(Q_y,z))+omega_y*(diff(Q_x,z)-diff(Q_z,x))
+omega_z*(diff(Q_y,x)-diff(Q_x,y));
```

$$\begin{aligned}
(\%o14) \quad & Q_x \left(\frac{d}{dy} \omega_z - \frac{d}{dz} \omega_y \right) + Q_y \left(\frac{d}{dz} \omega_x - \frac{d}{dx} \omega_z \right) + Q_z \left(\frac{d}{dx} \omega_y - \frac{d}{dy} \omega_x \right) = \\
& \left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x \right) \omega_z + \left(\frac{d}{dz} Q_x - \frac{d}{dx} Q_z \right) \omega_y + \left(\frac{d}{dy} Q_z - \frac{d}{dz} Q_y \right) \omega_x
\end{aligned}$$

```
(%i15) feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0) := (
    print("E15a: ", R1: ev(E15a,diff)),
    print("E15b: ", R2: ev(E15b,diff)),
    print("E15c: ", ev(E15c,diff)),
    print("E19: ", ev(E19,diff)),
    print("E20: ", ev(E20,diff)),
    print("E21: ", ev(E21,diff)),
    print("E23: ", ev(E23,diff)), ""
);

(%o15) feval(Q_x, Q_y, Q_z, omega_x, omega_y, omega_z, omega_0) := (
print(E15a: , R1: ev(E15a, diff)), print(E15b: , R2: ev(E15b, diff)),
print(E15c: , ev(E15c, diff)), print(E19: , ev(E19, diff)),
print(E20: , ev(E20, diff)), print(E21: , ev(E21, diff)),
print(E23: , ev(E23, diff)), )
```

□ 2 Special cases

□ 2.1 Q

```
(%i16) kill(omega_0,omega_x,omega_y,omega_z,Q_x,Q_y,Q_z,Phi,Q_0);
(%o16) done
```

```
(%i17) depends([omega_0],[t],[omega_x,omega_y,omega_z,Q_x,Q_y,Q_z,Phi], [x,
(%o17) [omega_0(t), omega_x(x,y,z), omega_y(x,y,z), omega_z(x,y,z), Q_x(x,y,z), Q_y(x,y,z),
Q_z(x,y,z), Phi(x,y,z)]
```

```
(%i20) Q_x: Q_0*sin(%beta*t-(k_x*x+k_y*y+k_z*z));
Q_y: 0;
Q_z: 0;

(%o18) -Q_0 sin(k_z z+k_y y+k_x x-beta t)
(%o19) 0
(%o20) 0
```

```
(%i21) feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0);
E15a: 0=0
E15b: beta Q_0 omega_z cos(k_z z+k_y y+k_x x-beta t)=Q_0 k_z omega_0 cos(k_z z+k_y y+k_x x-beta t)
E15c: -beta Q_0 omega_y cos(k_z z+k_y y+k_x x-beta t)=-Q_0 k_y omega_0 cos(k_z z+k_y y+k_x x-beta t)

E19: 0=0
E20: Q_0 omega_z sin(k_z z+k_y y+k_x x-beta t)-Q_0 k_z cos(k_z z+k_y y+k_x x-beta t)=0
E21: 0=Q_0 k_y cos(k_z z+k_y y+k_x x-beta t)-Q_0 omega_y sin(k_z z+k_y y+k_x x-beta t)
E23: -Q_0 (d/d y omega_z - d/d z omega_y) sin(k_z z+k_y y+k_x x-beta t)=Q_0 k_y omega_z
cos(k_z z+k_y y+k_x x-beta t)-Q_0 k_z omega_y cos(k_z z+k_y y+k_x x-beta t)
(%o21)
```

```
(%i22) ratsimp(R2/Q_0/cos(k_z*z+k_y*y+k_x*x-%beta*t));
(%o22)  $\beta \omega_z = k_z \omega_0$ 
```

□ 2.2 omega=const.

```
(%i23) kill(omega_0, omega_x, omega_y, omega_z, Q_x, Q_y, Q_z, Phi, Q_0);
(%o23) done
```

```
(%i24) depends([omega_0],[t],[omega_x,omega_y,omega_z,Q_x,Q_y,Q_z,Phi],[x,
(%o24) [ $\omega_0(t)$ ,  $\omega_x(x,y,z,t)$ ,  $\omega_y(x,y,z,t)$ ,  $\omega_z(x,y,z,t)$ ,  $Q_x(x,y,z,t)$ ,
 $Q_y(x,y,z,t)$ ,  $Q_z(x,y,z,t)$ ,  $\Phi(x,y,z,t)$ ]
```

```
(%i27) omega_x: 0;
      omega_y: 0;
      omega_z: kappa;
(%o25) 0
(%o26) 0
(%o27)  $\kappa$ 
```

```
(%i28) feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0);
E15a:  $-\left(\frac{d}{dt} Q_y\right) \kappa = -\left(\frac{d}{dy} Q_z - \frac{d}{dz} Q_y\right) \omega_0$ 
E15b:  $\left(\frac{d}{dt} Q_x\right) \kappa = -\left(\frac{d}{dz} Q_x - \frac{d}{dx} Q_z\right) \omega_0$ 
E15c:  $0 = -\left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x\right) \omega_0$ 
E19:  $\frac{d}{dy} Q_z = Q_y \kappa - \frac{d}{dz} Q_y$ 
E20:  $\frac{d}{dz} Q_x - Q_x \kappa = -\frac{d}{dx} Q_z$ 
E21:  $\frac{d}{dx} Q_y = -\frac{d}{dy} Q_x$ 
E23:  $0 = \left(\frac{d}{dx} Q_y - \frac{d}{dy} Q_x\right) \kappa$ 
(%o28)
```

□ 2.3 Additionally: $Q_x = Q_y = f(t)$ only

```
(%i31) Q_x: q*sin(%beta*t);
      Q_y: q*cos(%beta*t);
      Q_z: a(t)*x+b(t)*y;
(%o29)  $q \sin(\beta t)$ 
(%o30)  $q \cos(\beta t)$ 
(%o31)  $b(t) y + a(t) x$ 
```

```

(%i32) feval(Q_x,Q_y,Q_z,omega_x,omega_y,omega_z,omega_0);
E15a:  $\beta \kappa q \sin(\beta t) = -\omega_0 b(t)$ 
E15b:  $\beta \kappa q \cos(\beta t) = \omega_0 a(t)$ 
E15c:  $0 = 0$ 
E19:  $b(t) = \kappa q \cos(\beta t)$ 
E20:  $-\kappa q \sin(\beta t) = -a(t)$ 
E21:  $0 = 0$ 
E23:  $0 = 0$ 
(%o32)

```

```

(%i33) Omega: curl([Q_x,Q_y,Q_z])-cross([0,0,kappa],[Q_x,Q_y,Q_z]);
(%o33) [ $\kappa q \cos(\beta t) + b(t)$ ,  $-\kappa q \sin(\beta t) - a(t)$ , 0]

```

```

(%i34) ev(Omega, [a(t)=kappa*q*cos(%beta*t), b(t)=-kappa*q*sin(%beta*t)]);
(%o34) [ $\kappa q \cos(\beta t) - \kappa q \sin(\beta t)$ ,  $-\kappa q \sin(\beta t) - \kappa q \cos(\beta t)$ , 0]

```

```

(%i35) Omega: (-sqrt(2)*kappa*q*[sin(%beta*t-%pi/4), sin(%beta*t+%pi/4), 0]
(%o35) [ $-\sqrt{2} \kappa q \sin\left(\beta t - \frac{\pi}{4}\right)$ ,  $-\sqrt{2} \kappa q \sin\left(\beta t + \frac{\pi}{4}\right)$ , 0]

```