Orbital precession from the Minkowski metric in ECE2 theory

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3 Additional analysis

In Einsteinian theory the orbit $\theta(u)$ with $u = 1/r$ has to be computed from solving the integral

$$\theta(u) = \int \frac{L_0\, du}{\sqrt{2m(H + ku - \frac{L_0^2}{2m}u^2 + \frac{L_0^2}{2m}r_0 u^3)}} \tag{57}$$

with non-relativistic angular momentum L_0, total energy H, $k = mMG$ and "Schwarzschild radius" r_0. The term in the square root is a polynomial of third order in u and can be written as

$$\frac{1}{\alpha}(u - u_1)(u - u_2)(u - u_3) \tag{58}$$

where $u_1 = 1/r_1$ etc. are characteristic inverse radii. The constants u_1, u_2, u_3 are defined by Eq.(57), and

$$\frac{1}{\alpha} = u_1 + u_2 + u_3. \tag{59}$$

Einstein argued by the roots of Eq.(58). The physical range of u is between two values of u where the denominator vanishes, i.e. one has to find the roots of (58) to find the integration interval. In his terminology Einstein wrote the terms in the denominator in form of

$$\frac{2A}{B^2} + \frac{\alpha}{B^2}u - u^2 + \alpha u^3 \tag{60}$$

and additionally omitted the cubic term. This seems to be arbitrary but guarantees that only two roots exist which then are

$$u^{(1,2)} = \frac{\pm\sqrt{8\,A\,B^2 + \alpha^2} + \alpha}{2\,B^2}. \tag{61}$$

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The correct method, however, would be finding the roots of the cubic equation (58). By computer algebra this is possible. Quite complicated solutions follow from which two are complex-valued. This problem of the "true" solution of (58) have never been addressed in literature.

With modern computer algebra, it is possible to solve Eq.(57) analytically. Writing it in the form

$$\theta(u) = \int \frac{du}{\sqrt{\alpha(u-u_1)(u-u_2)(u-u_3)}} \tag{62}$$

leads to a solution which, after some simplifications, reads

$$\theta(u) = \frac{2}{\sqrt{\alpha(u_2-u_1)}} \, \mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{u_1-u_2}{u_1-u}}\right), \, \frac{u_3-u_1}{u_2-u_1}\right) \tag{63}$$

with the elliptic integral of first kind $\mathrm{F}(x,y)$. It has to be noted that this integral is complex-valued. The real value has to be taken as physical value.

Having found this solution, the result can be plotted and computer graphics gives an impression of the solution immediately. First we have graphed the integrand of (62) as a function $f(u)$ with parameters $u_1 = 3,\ u_2 = 2,\ \alpha = 0.1$ from which follows $u_3 = 5$. Fig. 1 shows that the integrand has strong infinite asymptotes as was already known from corresponding plots in UFT papers 150 and 155. u_1 and u_2 are the physical inverse radii, above u_3 an unlimited unphysical range appears. The real part of solution (63) (Fig. 2) is dominated by the inverse sine function which is defined between u_1 and u_2 correctly. The imaginary part pertains to an unphysical range. Choosing parameters differently with $u_1 < u_2$ (not shown) gives similar results with positive $\theta(u)$. We conclude that there is no multiplicity of solution for θ, i.e. there is no room for any precession effects from this Einsteinian solution which probably was analysed in these details for the first time.

The last example is an assessment of relativistic effects for a non-relativistic elliptic orbit. The latter is given by

$$r = \frac{\alpha}{1+\epsilon\cos(\theta)}. \tag{64}$$

We assume that the half-right latitude α is affected by relativistic effects:

$$\alpha = \gamma\ \alpha_0 = \frac{1}{1-v_0^2/c^2}\,\alpha_0 \tag{65}$$

for a non-relativistic α_0. Using the well-known solution

$$v_0^2 = \left(\frac{2}{r} - \frac{1}{a}\right) MG \tag{66}$$

and inserting this into (64), we obtain an equation for the orbit $r(\theta)$ with relativistic correction:

$$r = \frac{(2\,a\,\epsilon\cos(\theta) + 2\,a)\ M\,G + a\,\alpha_0\,c^2}{(\epsilon\cos(\theta)+1)\ M\,G + a\,c^2\,\epsilon\cos(\theta) + a\,c^2}. \tag{67}$$

The graph (Fig. 3) shows what is to be expected from (65): the effective alpha is enlarged by relativistic effects (here obtained by varying c and keeping all other parameters to unity). The enlargement is not constant, but there is no crossing of the curves, that means that the constants of motion are different. This is plausible because the angular momentum L_0 is increased by the gamma factor. A smaller c here means stronger relativistic effects.

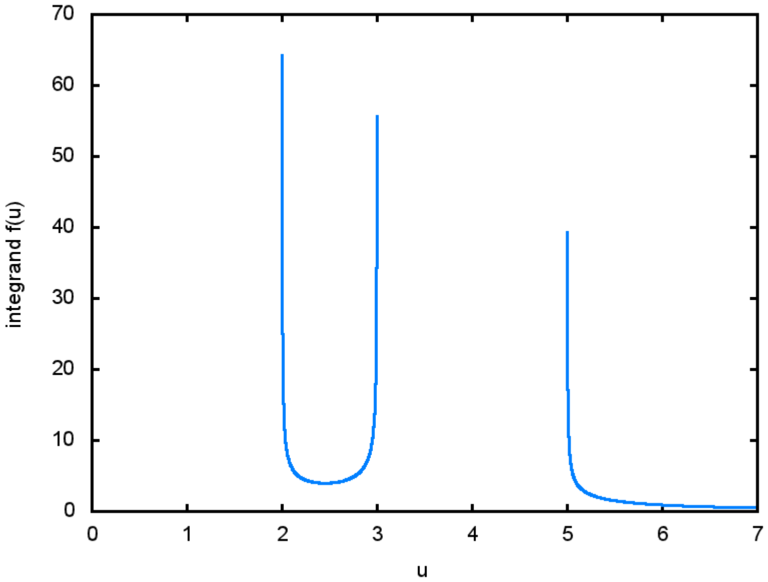

Figure 1: Integrand of Einstein integral in form of Eq.(62).

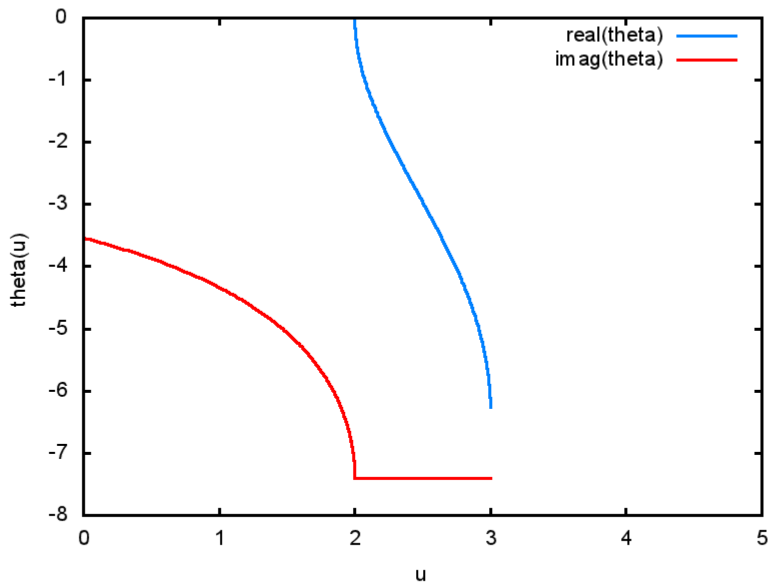

Figure 2: Analytical solution (63) of the Einstein integral.

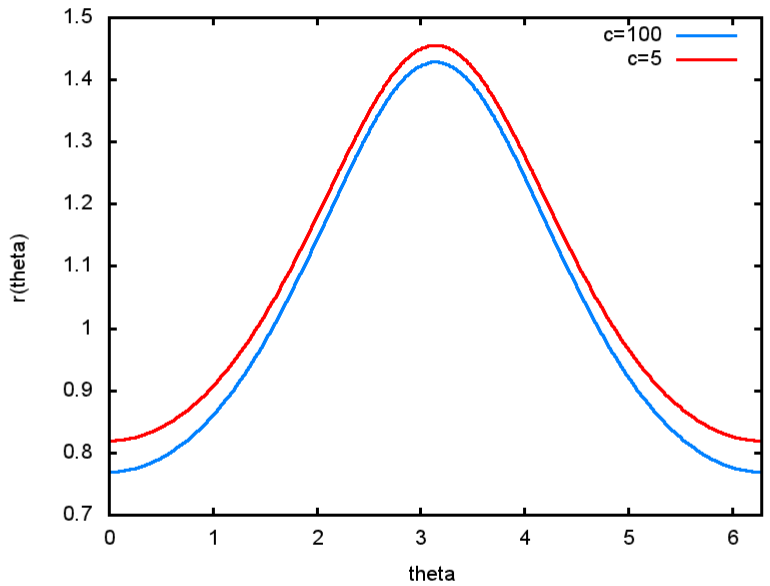

Figure 3: Radius function $r(\theta)$ for different cases of relativistic effects, characterized by c.