```
(%i22) kill(all);
(%o0) done
```

1 Potential

```
(%i1) U: -e^2/(4*%pi*epsilon[0]*r);
```

$$(\%o1)\quad -\frac{e^2}{4\,\pi\,\varepsilon_0\,r}$$

2 Constants

```
(%i2) a[0]: 4*%pi*epsilon[0]*h[bar]^2/(m*e^2);
```

$$(\%o2)\quad \frac{4\,\pi\,\varepsilon_0\,h_{bar}^2}{e^2\,m}$$

```
(%i3) alpha: 1/(4*%pi*epsilon[0]*c)*e^2/h[bar];
```

$$(\%o3)\quad \frac{e^2}{4\,\pi\,\varepsilon_0\,h_{bar}\,c}$$

```
(%i4) Ry: -m*e^4/(32*%pi^2*epsilon[0]^2*h[bar]^2)*1/n^2;
```

$$(\%o4)\quad -\frac{e^4\,m}{32\,\pi^2\,\varepsilon_0^2\,h_{bar}^2\,n^2}$$

3 Eq.(10)

```
(%i5) U10: -h[bar]*c*alpha/a[0];
```

$$(\%o5)\quad -\frac{e^4\,m}{16\,\pi^2\,\varepsilon_0^2\,h_{bar}^2}$$

```
(%i6) EU: -e^2/(4*%pi*epsilon[0]*a[0]);
```

$$(\%o6)\quad -\frac{e^4\,m}{16\,\pi^2\,\varepsilon_0^2\,h_{bar}^2}$$

```
(%i7) U10-EU;
(%o7) 0
```

4 Eq.(12)

```
(%i8) H1: -e/(2*m)*(1+1/2*p02/(m^2*c^2))*L[0]*B;
```

$$(\%o8)\quad -\frac{L_0\,e\left(\frac{p02}{2\,c^2\,m^2}+1\right)B}{2\,m}$$

4.1 From (11):

```
(%i9) p02: 2*m*h[bar]*c*alpha/a[0]/(2*n^2);
```

$$(\%o9)\quad \frac{e^4\,m^2}{16\,\pi^2\,\varepsilon_0^2\,h_{bar}^2\,n^2}$$

4.2 From (7):

```
(%i10) L[0]: h[bar]*m[L];
```

$$(\%o10)\quad h_{bar}\,m_L$$

4.3 Result (12):

```
(%i11) "<H1>" = ev(H1);
```

$$(\%o11)\quad \langle H1\rangle = -\frac{h_{bar}\,e\left(\frac{e^4}{32\,\pi^2\,\varepsilon_0^2\,h_{bar}^2\,c^2\,n^2}+1\right)B\,m_L}{2\,m}$$

5 Eq.(13)

```
(%i12) H01: 1/2*lambda[C]*alpha/(n^2*a[0]);
```

$$(\%o12)\quad \frac{e^4\,m\,lambda_C}{32\,\pi^2\,\varepsilon_0^2\,h_{bar}^3\,c\,n^2}$$

```
(%i13) lambda[C]: h[bar]/(m*c);
```

$$(\%o13)\quad \frac{h_{bar}}{c\,m}$$

```
(%i14) ev(H01);
```

$$(\%o14)\quad \frac{e^4}{32\,\pi^2\,\varepsilon_0^2\,h_{bar}^2\,c^2\,n^2}$$

6 Numerical factor in (18)

```
(%i15) lambda[C]: 3.861591e-13;
       a[0]: 5.29177e-11;
       alpha: 0.007297351;
```

$$(\%o15)\quad 3.8615909999999998\,10^{-13}$$

$$(\%o16)\quad 5.29177\,10^{-11}$$

$$(\%o17)\quad 0.007297351$$

```
(%i18) 1/2*lambda[C]*alpha/(n^2*a[0]);
```

$$(\%o18)\ \frac{2.6625670565274942\ 10^{-5}}{n^2}$$