```
(%i1) kill(all);
(%o0) done
```

(%i1) batchload("D:/Doc/Artikel-Eck/ECE-Theorie/Paper253/RY-Hydrogen.wxm

0 : $$-\frac{2\,Z^{5/2}\,\%e^{-\frac{r\,Z}{a_0}}}{a_0^{5/2}}$$

1 : $$\frac{\sqrt{Z}\,(\sqrt{2}\,r\,Z^3-2^{5/2}\,a_0\,Z^2)\,\%e^{-\frac{r\,Z}{2\,a_0}}}{8\,a_0^{7/2}}$$

2 : $$-\frac{\sqrt{Z}\,(\sqrt{6}\,r\,Z^3-2\,\sqrt{6}\,a_0\,Z^2)\,\%e^{-\frac{r\,Z}{2\,a_0}}}{24\,a_0^{7/2}}$$

3 : $$-\frac{\sqrt{Z}\,(4\,\sqrt{3}\,r^2\,Z^4-20\,3^{3/2}\,a_0\,r\,Z^3+2\,3^{9/2}\,a_0^2\,Z^2)\,\%e^{-\frac{r\,Z}{3\,a_0}}}{729\,a_0^{9/2}}$$

4 : $$\frac{\sqrt{Z}\,(4\,r^2\,Z^4-48\,a_0\,r\,Z^3+72\,a_0^2\,Z^2)\,\%e^{-\frac{r\,Z}{3\,a_0}}}{243\,\sqrt{6}\,a_0^{9/2}}$$

5 : $$-\frac{\sqrt{Z}\,(2\,\sqrt{30}\,r^2\,Z^4-12\,\sqrt{30}\,a_0\,r\,Z^3)\,\%e^{-\frac{r\,Z}{3\,a_0}}}{3645\,a_0^{9/2}}$$

0 : $$\frac{2\,Z^{7/2}\,\%e^{-\frac{r\,Z}{a_0}}}{a_0^{7/2}}$$

1 : $$-\frac{\sqrt{Z}\,(r\,Z^4-6\,a_0\,Z^3)\,\%e^{-\frac{r\,Z}{2\,a_0}}}{2^{7/2}\,a_0^{9/2}}$$

2 : $$\frac{\sqrt{Z}\,(r\,Z^4-4\,a_0\,Z^3)\,\%e^{-\frac{r\,Z}{2\,a_0}}}{8\,\sqrt{6}\,a_0^{9/2}}$$

3 : $$\frac{\sqrt{Z}\,(4\,\sqrt{3}\,r^2\,Z^5-28\,3^{3/2}\,a_0\,r\,Z^4+38\,3^{5/2}\,a_0^2\,Z^3)\,\%e^{-\frac{r\,Z}{3\,a_0}}}{2187\,a_0^{11/2}}$$

4 : $$-\frac{\sqrt{Z}\,(2\,\sqrt{6}\,r^2\,Z^5-6^{5/2}\,a_0\,r\,Z^4+3\,6^{5/2}\,a_0^2\,Z^3)\,\%e^{-\frac{r\,Z}{3\,a_0}}}{2187\,a_0^{11/2}}$$

5 : $$\frac{\sqrt{Z}\,(2\,\sqrt{30}\,r^2\,Z^5-24\,\sqrt{30}\,a_0\,r\,Z^4+36\,\sqrt{30}\,a_0^2\,Z^3)\,\%e^{-\frac{r\,Z}{3\,a_0}}}{10935\,a_0^{11/2}}$$

0 N(R): 1 N(dR): $\frac{Z^2}{a_0^2}$ N(d2R): $\frac{Z^4}{a_0^4}$

1 N(R): 1 N(dR): $\frac{Z^2}{4\,a_0^2}$ N(d2R): $\frac{3\,Z^4}{16\,a_0^4}$

2 N(R): 1 N(dR): $\frac{Z^2}{12\,a_0^2}$ N(d2R): $\frac{Z^4}{48\,a_0^4}$

3 N(R): 1 N(dR): $\frac{Z^2}{9\,a_0^2}$ N(d2R): $\frac{49\,Z^4}{729\,a_0^4}$

4 N(R): 1 N(dR): $\frac{5\,Z^2}{36\,a_0^2}$ N(d2R): $\frac{Z^4}{}$

1 Eq.(27)

```
(%i2) Hso2: -e*h[bar]*U/(4*m^2*c^2)*sigma*B;
```

$$(\%o2)\quad -\frac{h_{bar}\, e\, \sigma\, B\, U}{4\, c^2\, m^2}$$

```
(%i3) U: -e^2/(4*%pi*epsilon[0]*r);
```

$$(\%o3)\quad -\frac{e^2}{4\,\pi\,\varepsilon_0\, r}$$

```
(%i4) ev(Hso2);
```

$$(\%o4)\quad \frac{h_{bar}\, e^3\, \sigma\, B}{16\,\pi\,\varepsilon_0\, c^2\, m^2\, r}$$

2 Eq.(30)

2.1 <U>

```
(%i5) U;
```

$$(\%o5)\quad -\frac{e^2}{4\,\pi\,\varepsilon_0\, r}$$

```
(%i6) i: 0$
      for n: 1 thru 3 do (
       for l: 0 thru n-1 do (
        for ml: 0 thru l do (
          EU: Ex3(psi[i], U),
          if equal(n,1) then E_U: EU,
          printf(true, "~s EU=~a ~%", qn[i], EU),
          i: i+1
      )))$
"n=1, l=0, ml=0" EU=-e^2*Z/(4*%pi*a[0]*epsilon[0])
"n=2, l=0, ml=0" EU=-e^2*Z/(16*%pi*a[0]*epsilon[0])
"n=2, l=1, ml=0" EU=-e^2*Z/(16*%pi*a[0]*epsilon[0])
"n=2, l=1, ml=1" EU=-e^2*Z/(16*%pi*a[0]*epsilon[0])
"n=3, l=0, ml=0" EU=-e^2*Z/(36*%pi*a[0]*epsilon[0])
"n=3, l=1, ml=0" EU=-e^2*Z/(36*%pi*a[0]*epsilon[0])
"n=3, l=1, ml=1" EU=-e^2*Z/(36*%pi*a[0]*epsilon[0])
"n=3, l=2, ml=0" EU=-e^2*Z/(36*%pi*a[0]*epsilon[0])
"n=3, l=2, ml=1" EU=-e^2*Z/(36*%pi*a[0]*epsilon[0])
"n=3, l=2, ml=2" EU=-e^2*Z/(36*%pi*a[0]*epsilon[0])
```

2.2 Bohr radius

(%i8) a[0]: 4*%pi*epsilon[0]*h[bar]^2/(m*e^2);

(%o8) $$\frac{4\,\pi\,\varepsilon_0\,h_{bar}^2}{e^2\,m}$$

(%i9) E_U: ev(E_U);

(%o9) $$-\frac{e^4\,m\,Z}{16\,\pi^2\,\varepsilon_0^2\,h_{bar}^2}$$

2.3 <U> and <Hso2>

(%i10) U: E_U/n^2;

(%o10) $$-\frac{e^4\,m\,Z}{16\,\pi^2\,\varepsilon_0^2\,h_{bar}^2\,n^2}$$

(%i11) ev(Hso2);

(%o11) $$\frac{e^5\,\sigma\,B\,Z}{64\,\pi^2\,\varepsilon_0^2\,h_{bar}\,c^2\,m\,n^2}$$