```
(%i1) kill(all);
(%o0) done
```

```
(%i1) E24: p^2/m = (1+gamma)*H1;
```

$$(\%o1)\quad \frac{p^2}{m}=H1(\Gamma+1)$$

```
(%i2) v: p/m;
```

$$(\%o2)\quad \frac{p}{m}$$

```
(%i3) gamma: 1/sqrt(1-v^2/c^2);
```

$$(\%o3)\quad \frac{1}{\sqrt{1-\frac{p^2}{c^2\,m^2}}}$$

```
(%i4) E33: ev(E24);
```

$$(\%o4)\quad \frac{p^2}{m}=\left(\frac{1}{\sqrt{1-\frac{p^2}{c^2\,m^2}}}+1\right)H1$$

1 Exact relativistic equation

```
(%i5) assume (m>0, c>0);
(%o5) [m>0,c>0]
```

```
(%i6) E33a: ratsimp(E33^2);
```

$$(\%o6)\quad \frac{p^4}{m^2}=-\frac{2\,c\,m\sqrt{c^2\,m^2-p^2}\,H1^2+(2\,c^2\,m^2-p^2)H1^2}{p^2-c^2\,m^2}$$

```
(%i7) E33b: E33a*(p^2-c^2*m^2);
```

$$(\%o7)\quad \frac{p^4(p^2-c^2\,m^2)}{m^2}=-2\,c\,m\sqrt{c^2\,m^2-p^2}\,H1^2-(2\,c^2\,m^2-p^2)H1^2$$

```
(%i8) E33c: E33b + (2*c^2*m^2-p^2)*H1^2;
```

$$(\%o8)\quad (2\,c^2\,m^2-p^2)H1^2+\frac{p^4(p^2-c^2\,m^2)}{m^2}=-2\,c\,m\sqrt{c^2\,m^2-p^2}\,H1^2$$

```
(%i9) E33d: (E33c^2);
```

$$(\%o9)\quad \left((2\,c^2\,m^2-p^2)H1^2+\frac{p^4(p^2-c^2\,m^2)}{m^2}\right)^2=4\,c^2\,m^2(c^2\,m^2-p^2)H1^4$$

```
(%i10) E33e: ratsubst(p2, p^2, E33d);
```

$$(\%o10)\quad ((m^4\,p2^2-4\,c^2\,m^6\,p2+4\,c^4\,m^8)H1^4+(-2\,m^2\,p2^4+6\,c^2\,m^4\,p2^3-4\,c^4\,m^6\,p2^2)H1^2+p2^6-2\,c^2\,m^2\,p2^5+c^4\,m^4\,p2^4)/m^4=(4\,c^4\,m^4-4\,c^2\,m^2\,p2)H1^4$$

(%i11) `E33f: solve(E33e, p2);`

(%o11) $$[p2=-\frac{-2\,m\,H1+c\,m^{3/2}\sqrt{c^2\,m-4\,H1}-c^2\,m^2}{2},\ p2=\frac{2\,m\,H1+c\,m^{3/2}\sqrt{c^2\,m-4\,H1}+c^2\,m^2}{2},\ p2=-\frac{c\,m^{3/2}\sqrt{4\,H1+c^2\,m}+2\,m\,H1-c^2\,m^2}{2},\ p2=\frac{c\,m^{3/2}\sqrt{4\,H1+c^2\,m}-2\,m\,H1+c^2\,m^2}{2},\ p2=0]$$

2 Solutions for p^2

(%i12) `S1: ratsimp(first(E33f));`

(%o12) $$p2=-\frac{-2\,m\,H1+c\,m^{3/2}\sqrt{c^2\,m-4\,H1}-c^2\,m^2}{2}$$

(%i13) `S2: ratsimp(second(E33f));`

(%o13) $$p2=\frac{2\,m\,H1+c\,m^{3/2}\sqrt{c^2\,m-4\,H1}+c^2\,m^2}{2}$$

(%i14) `S3: ratsimp(third(E33f));`

(%o14) $$p2=-\frac{c\,m^{3/2}\sqrt{4\,H1+c^2\,m}+2\,m\,H1-c^2\,m^2}{2}$$

(%i15) `S4: ratsimp(fourth(E33f));`

(%o15) $$p2=\frac{c\,m^{3/2}\sqrt{4\,H1+c^2\,m}-2\,m\,H1+c^2\,m^2}{2}$$