```
(%i21) kill(all);
(%o0) done
```

1 Eq.(10)

```
(%i1) L1: matrix([gamma,0,0,-beta*gamma],
      [0,1,0,0],
      [0,0,1,0],
      [-beta*gamma,0,0,gamma]);
```

(%o1)
$$\begin{bmatrix} \Gamma & 0 & 0 & -\mathit{beta}\,\Gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathit{beta}\,\Gamma & 0 & 0 & \Gamma \end{bmatrix}$$

2 Transformation in Z direction

```
(%i2) J4: transpose([c*rho, J[x], J[y], J[z]]);
```

(%o2)
$$\begin{bmatrix} c\,\rho \\ J_x \\ J_y \\ J_z \end{bmatrix}$$

```
(%i3) LJ: factor(L1.J4);
```

(%o3)
$$\begin{bmatrix} -(\mathit{beta}\,J_z - c\,\rho)\,\Gamma \\ J_x \\ J_y \\ (J_z - \mathit{beta}\,c\,\rho)\,\Gamma \end{bmatrix}$$

```
(%i4) A4: transpose([phi/c, A[x], A[y], A[z]]);
```

(%o4)
$$\begin{bmatrix} \frac{\phi}{c} \\ A_x \\ A_y \\ A_z \end{bmatrix}$$

(%i5) LA: factor(L1.A4);

(%o5)
$$\begin{bmatrix} -\frac{(beta\ c\ A_z - \phi)\ \Gamma}{c} \\ A_x \\ A_y \\ \frac{(c\ A_z - beta\ \phi)\ \Gamma}{c} \end{bmatrix}$$

3 Transformation in general direction

(%i6) Lambda: matrix([gamma, -gamma*b[x], -gamma*b[y], -gamma*b[z]],
[-gamma*b[x], 1+(gamma-1)*(b[x]/b[0])^2, (gamma-1)*b[x]*b[y]/b[0]^
(gamma-1)*b[x]*b[z]/b[0]^2],
[-gamma*b[y], (gamma-1)*b[y]*b[x]/b[0]^2, 1+(gamma-1)*(b[y]/b[0])^
(gamma-1)*b[y]*b[z]/b[0]^2],
[-gamma*b[z], (gamma-1)*b[x]*b[z]/b[0]^2, (gamma-1)*b[z]*b[y]/b[0]
1+(gamma-1)*(b[z]/b[0])^2]);

(%o6)
$$\begin{bmatrix} \Gamma & -b_x\,\Gamma & -b_y\,\Gamma & -b_z\,\Gamma \\ -b_x\,\Gamma & \frac{b_x^2\,(\Gamma-1)}{b_0^2}+1 & \frac{b_x\,b_y\,(\Gamma-1)}{b_0^2} & \frac{b_x\,b_z\,(\Gamma-1)}{b_0^2} \\ -b_y\,\Gamma & \frac{b_x\,b_y\,(\Gamma-1)}{b_0^2} & \frac{b_y^2\,(\Gamma-1)}{b_0^2}+1 & \frac{b_y\,b_z\,(\Gamma-1)}{b_0^2} \\ -b_z\,\Gamma & \frac{b_x\,b_z\,(\Gamma-1)}{b_0^2} & \frac{b_y\,b_z\,(\Gamma-1)}{b_0^2} & \frac{b_z^2\,(\Gamma-1)}{b_0^2}+1 \end{bmatrix}$$

(%i7) J4: transpose([c*rho, J[x], J[y], J[z]]);

(%o7)
$$\begin{bmatrix} c\ \rho \\ J_x \\ J_y \\ J_z \end{bmatrix}$$

(%i8) LJ: (Lambda.transpose(J4));

(%o8)
$$\begin{bmatrix} -b_z\,J_z\,\Gamma - b_y\,J_y\,\Gamma - b_x\,J_x\,\Gamma + c\,\rho\,\Gamma \\ -c\,\rho\,b_x\,\Gamma + \frac{b_x\,b_z\,J_z\,(\Gamma-1)}{b_0^2} + \frac{b_x\,b_y\,J_y\,(\Gamma-1)}{b_0^2} + J_x\left(\frac{b_x^2\,(\Gamma-1)}{b_0^2}+1\right) \\ -c\,\rho\,b_y\,\Gamma + \frac{b_y\,b_z\,J_z\,(\Gamma-1)}{b_0^2} + \frac{b_x\,J_x\,b_y\,(\Gamma-1)}{b_0^2} + J_y\left(\frac{b_y^2\,(\Gamma-1)}{b_0^2}+1\right) \\ -c\,\rho\,b_z\,\Gamma + \frac{b_y\,J_y\,b_z\,(\Gamma-1)}{b_0^2} + \frac{b_x\,J_x\,b_z\,(\Gamma-1)}{b_0^2} + J_z\left(\frac{b_z^2\,(\Gamma-1)}{b_0^2}+1\right) \end{bmatrix}$$

(%i9) A4: transpose([phi/c, A[x], A[y], A[z]]);

(%o9)
$$\begin{bmatrix} \frac{\phi}{c} \\ A_x \\ A_y \\ A_z \end{bmatrix}$$

(%i10) LA: (Lambda.A4);

(%o10)
$$\begin{bmatrix} -b_z A_z \Gamma - b_y A_y \Gamma - b_x A_x \Gamma + \frac{\phi \Gamma}{c} \\ -\frac{\phi b_x \Gamma}{c} + \frac{b_x b_z A_z (\Gamma - 1)}{b_0^2} + \frac{b_x b_y A_y (\Gamma - 1)}{b_0^2} + A_x \left(\frac{b_x^2 (\Gamma - 1)}{b_0^2} + 1 \right) \\ -\frac{\phi b_y \Gamma}{c} + \frac{b_y b_z A_z (\Gamma - 1)}{b_0^2} + \frac{b_x A_x b_y (\Gamma - 1)}{b_0^2} + A_y \left(\frac{b_y^2 (\Gamma - 1)}{b_0^2} + 1 \right) \\ -\frac{\phi b_z \Gamma}{c} + \frac{b_y A_y b_z (\Gamma - 1)}{b_0^2} + \frac{b_x A_x b_z (\Gamma - 1)}{b_0^2} + A_z \left(\frac{b_z^2 (\Gamma - 1)}{b_0^2} + 1 \right) \end{bmatrix}$$

4 Reduction to special directions

4.1 Set beta_x = beta_y = 0

(%i11) b[x]: b[y]: 0;

(%o11) 0

(%i12) LJ: ev(Lambda.J4);

(%o12)
$$\begin{bmatrix} c \rho \Gamma - b_z J_z \Gamma \\ J_x \\ J_y \\ J_z \left(\frac{b_z^2 (\Gamma - 1)}{b_0^2} + 1 \right) - c \rho b_z \Gamma \end{bmatrix}$$

(%i13) LA: ev(Lambda.A4);

(%o13)
$$\begin{bmatrix} \frac{\phi \Gamma}{c} - b_z A_z \Gamma \\ A_x \\ A_y \\ A_z \left(\frac{b_z^2 (\Gamma - 1)}{b_0^2} + 1 \right) - \frac{\phi b_z \Gamma}{c} \end{bmatrix}$$

4.2 Compare with results for Lorentz transform in Z dir

(%i14)
```
L1J: matrix([-(beta*J[z]-c*rho)*gamma],[J[x]],[J[y]],[(J[z]-beta*c
```

(%o14)
$$\begin{bmatrix} (c\,\rho - beta\,J_z)\,\Gamma \\ J_x \\ J_y \\ (J_z - beta\,c\,\rho)\,\Gamma \end{bmatrix}$$

(%i15)
```
L1A:matrix([-((beta*c*A[z]-phi)*gamma)/c],[A[x]],[A[y]],[((c*A[z]-
```

(%o15)
$$\begin{bmatrix} -\dfrac{(beta\,c\,A_z - \phi)\,\Gamma}{c} \\ A_x \\ A_y \\ \dfrac{(c\,A_z - beta\,\phi)\,\Gamma}{c} \end{bmatrix}$$

(%i16)
```
beta: b[z]: v/c;
b[0]: v/c;
```

(%o16) $\dfrac{v}{c}$

(%o17) $\dfrac{v}{c}$

(%i18)
```
ratsimp(ev(LJ-L1J));
```

(%o18)
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(%i19)
```
ratsimp(ev(LA-L1A));
```

(%o19)
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$