```
(%i14) kill(all);
(%o0) done
```

```
(%i1) E12: 2*c^2*kappa[0]*kappa[1]*cos(theta[3]) = 2*omega[0]*omega[1]-x
```
$$(\%o1)\quad 2\,\kappa_0\,\kappa_1\,\cos(\theta_3)\,c^2=2\,\omega_0\,\omega_1-x$$

```
(%i2) E13: E12^2;
```
$$(\%o2)\quad 4\,\kappa_0^2\,\kappa_1^2\,\cos(\theta_3)^2\,c^4=(2\,\omega_0\,\omega_1-x)^2$$

```
(%i3) k0: sqrt((omega[0]/c)^2-x/c^2);
```
$$(\%o3)\quad \sqrt{\frac{\omega_0^2}{c^2}-\frac{x}{c^2}}$$

```
(%i4) E16a: ratsubst(k0, kappa[0], E13);
```
$$(\%o4)\quad 4\,\kappa_1^2\,\cos(\theta_3)^2\,c^4\left(\frac{\omega_0^2}{c^2}-\frac{x}{c^2}\right)=x^2-4\,\omega_0\,\omega_1\,x+4\,\omega_0^2\,\omega_1^2$$

```
(%i5) E16b: ratsubst(sqrt((omega[1]/c)^2-x/c^2), kappa[1], E16a);
```
$$(\%o5)\quad 4\,\omega_0^2\,\cos(\theta_3)^2\,c^2\left(\frac{\omega_1^2}{c^2}-\frac{x}{c^2}\right)-4\,\cos(\theta_3)^2\,c^2\,x\left(\frac{\omega_1^2}{c^2}-\frac{x}{c^2}\right)=x^2-4\,\omega_0\,\omega_1\,x+4\,\omega_0^2\,\omega_1^2$$

```
(%i6) E16: factor(E16b);
```
$$(\%o6)\quad 4\,\cos(\theta_3)^2\,(x-\omega_0^2)\,(x-\omega_1^2)=(x-2\,\omega_0\,\omega_1)^2$$

```
(%i7) E20: expand(E16);
```
$$(\%o7)\quad 4\,\cos(\theta_3)^2\,x^2-4\,\omega_1^2\,\cos(\theta_3)^2\,x-4\,\omega_0^2\,\cos(\theta_3)^2\,x+4\,\omega_0^2\,\omega_1^2\,\cos(\theta_3)^2=x^2-4\,\omega_0\,\omega_1\,x+4\,\omega_0^2\,\omega_1^2$$

```
(%i8) E23: solve(E20, x);
```
$$(\%o8)\quad [x=-(2\,\cos(\theta_3)\sqrt{(\omega_1^4-2\,\omega_0^2\,\omega_1^2+\omega_0^4)\cos(\theta_3)^2-2\,\omega_0\,\omega_1^3+5\,\omega_0^2\,\omega_1^2-2\,\omega_0^3\,\omega_1}+(-2\,\omega_1^2-2\,\omega_0^2)\cos(\theta_3)^2+2\,\omega_0\,\omega_1)/(4\,\cos(\theta_3)^2-1),\ x=(2\,\cos(\theta_3)\sqrt{(\omega_1^4-2\,\omega_0^2\,\omega_1^2+\omega_0^4)\cos(\theta_3)^2-2\,\omega_0\,\omega_1^3+5\,\omega_0^2\,\omega_1^2-2\,\omega_0^3\,\omega_1}+(2\,\omega_1^2+2\,\omega_0^2)\cos(\theta_3)^2-2\,\omega_0\,\omega_1)/(4\,\cos(\theta_3)^2-1)]$$

```
(%i9) assume(c>0, h[bar]>0);
```
$$(\%o9)\quad [c>0,\ h_{bar}>0]$$

```
(%i10) f: h[bar]*c^(-2);
```
$$(\%o10)\quad \frac{h_{bar}}{c^2}$$

(%i11) m1: (first(sqrt(E23)*h[bar]/c^2));

(%o11) $$\frac{h_{bar}\sqrt{x}}{c^2}=(sqrt(-(2\cos(\theta_3)\sqrt{(\omega_1^4-2\omega_0^2\omega_1^2+\omega_0^4)\cos(\theta_3)^2-2\omega_0\omega_1^3+5\omega_0^2\omega_1^2-2\omega_0^3\omega_1}+(-2\omega_1^2-2\omega_0^2)\cos(\theta_3)^2+2\omega_0\omega_1)/(4\cos(\theta_3)^2-1))h_{bar})/c^2$$

(%i12) m2: (second(sqrt(E23)*h[bar]/c^2));

(%o12) $$\frac{h_{bar}\sqrt{x}}{c^2}=(sqrt((2\cos(\theta_3)\sqrt{(\omega_1^4-2\omega_0^2\omega_1^2+\omega_0^4)\cos(\theta_3)^2-2\omega_0\omega_1^3+5\omega_0^2\omega_1^2-2\omega_0^3\omega_1}+(2\omega_1^2+2\omega_0^2)\cos(\theta_3)^2-2\omega_0\omega_1)/(4\cos(\theta_3)^2-1))h_{bar})/c^2$$