GENERAL THEORY AND CLASSIFICATION OF THREE DIMENSIONAL ORBITS.

by

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ABSTRACT

The general theory of three dimensional orbits is developed for any potential of attraction between an orbiting mass m and an attracting mass M. In general any three dimensional orbit can be constructed from the beta conic section, and classified in terms of ellipticity. In Cartesian representation there are sixteen classes of orbit, representing three dimensional conic sections. The theory is illustrated with the three dimensional whirlpool galaxy, and equations are developed for the animation of three dimensional orbits.

Keywords: ECE theory, three dimensional orbits, general theory and classification.

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1. INTRODUCTION.

In recent papers of this series {1 - 10} the theory of three dimensional orbits has been developed by replacing the plane polar coordinates with spherical polar coordinates in the kinetic energy. This procedure has resulted in a large number of novel results in astronomy. In general the hamiltonian can be represented by the beta conic section, where beta is defined in terms of the angles ϕ and θ of the spherical polar coordinate system of coordinates. There are four orbital functions in general, r can be a function of beta, theta, phi and a three dimensional combination of theta and phi. In two dimensional orbital theory r is a function of only of phi. In three dimensions there is more than one conserved angular momentum. The total angular momentum L is conserved, and the L_z component is conserved. In two dimensional theory only the L_z component is conserved. A three dimensional orbit r (ϕ, θ) can always be constructed from any potential of attraction U(r), so in general r is a function both of ϕ and θ. In two dimensional theory r is a function only of ϕ. For an inverse square law of attraction between an orbiting mass m and a central mass M, the three dimensional orbits can be deduced from a conic section in β for various ellipticities. In Cartesian representation it is shown in Section 2 that there are sixteen classifications of three dimensional orbit in general, equivalent to the three dimensional conic sections.

In Section 2 the general theory of three dimensional orbits is given for any potential of attraction U (r) and the theory illustrated with the hyperbolic spiral and logarithmic spiral orbits in three dimensions, these are examples of orbits generated with different types of inverse cubed force law of attraction. The inverse squared force law of attraction applied in three dimensions results in the three dimensional beta conic section. It is shown that there are sixteen fundamental Cartesian representations of the beta conic section in polar representation. Equations are given for the animation of three dimensional orbits.

In Section 3 the three dimensional whirlpool galaxy is graphed and discussed.

2. GENERAL THEORY AND CLASSIFICATIONS.

Consider the hamiltonian:

$$H = \frac{1}{2} m v^2 + U(r) \quad -(1)$$

and the lagrangian:

$$\mathcal{L} = \frac{1}{2} m v^2 - U(r) \quad -(2)$$

where U is any function of r. The solution of Eq. (1) is:

$$\frac{1}{r} = f(\beta) \quad -(3)$$

where f is any function of β . The force law equivalent to Eq. (1) is:

$$F(r) = -\frac{L^2}{m r^2}\left(\frac{d^2}{d\beta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) \quad -(4)$$

where:

$$m\ddot{r} = -\frac{L^2}{m r^2}\frac{d^2}{d\beta^2}\left(\frac{1}{r}\right). \quad -(5)$$

The transition from 2D to 3D orbital theory takes place through a transition in the kinetic energy:

$$T = \frac{1}{2} m v^2. \quad -(6)$$

The potential energy remains the same in 2D and 3D. In 2D:

$$v^2 = \dot{r}^2 + \dot{\phi}^2 r^2 \quad -(7)$$

and in 3D:

$$v^2 = \dot{r}^2 + r^2 \dot{\beta}^2 \quad -(8)$$

where:

$$\dot{\beta}^2 = \dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta. \quad -(9)$$

Eqs. (7) to (9) lead to {1 - 10}:

$$\tan\phi = \frac{L_Z}{L} \tan\beta \quad -(10)$$

and

$$\cos\theta = \left(1 - \left(\frac{L_Z}{L}\right)^2\right)^{1/2} \sin\beta. \quad -(11)$$

i.e.

$$\cos\beta = \frac{\cos\phi}{\left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi\right)^{1/2}} \quad -(12)$$

and

$$\cos\beta = \left(1 - \left(\frac{1}{1 - \left(\frac{L_Z}{L}\right)^2}\right) \cos^2\theta\right)^{1/2} \quad -(13)$$

In general, a three dimensional orbit is given by Eq. (3) with β defined by Eq. (12). Therefore r may be expressed in terms of ϕ, in terms of θ, and as a combination of both by adding Eqs. (12) and (13) to give:

$$\cos\beta = \frac{1}{2}\left[\frac{\cos\phi}{\left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi\right)^{1/2}} + \right.$$

$$\left(1-\left(\frac{1}{1-\left(\frac{L_Z}{L}\right)^2}\right)\cos^2\theta\right)^{1/2}\Bigg] . \quad -(14)$$

Eq. (14) gives and orbit r (ϕ, θ) for any β and any force law or potential U(r).

For example if the potential energy is:

$$U(r) = -\frac{L^2}{2m}\left(1+\gamma^2\right)\frac{1}{r^2} \quad -(15)$$

were γ is a constant, the force law is the inverse cubed:

$$F(r) = -\frac{L^2}{mr^3}\left(1+\gamma^2\right). \quad -(16)$$

Eq. (16) gives the logarithmic spiral beta orbit:

$$r = r_0 \exp\left(\gamma\beta\right). \quad -(17)$$

The lagrangian ($\mathcal{L}$) gives {1 - 10}:

$$\frac{d\beta}{dt} = \frac{L}{mr^2}. \quad -(18)$$

This equation gives the t dependence of β :

$$\beta(t) = \frac{1}{2\gamma}\log_e\left(\frac{2\gamma L t}{m r_0^2}\right) \quad -(19)$$

and of r:

$$r(t) = \left(\frac{2\gamma L t}{m}\right)^{1/2}. \quad -(20)$$

Therefore from Eqs. (10) and (11):

$$\phi(t) = \tan^{-1}\left(\frac{L_Z}{L}\tan\beta(t)\right) \quad -(21)$$

$$\theta(t) = \cos^{-1}\left(\left(1-\left(\frac{L_Z}{L}\right)^2\right)^{1/2}\sin\beta(t)\right). \quad -(22)$$

The three dimensional time dependence of the orbit is found from Eqs. (14), (21) and (22).

The three dimensional hyperbolic spiral is defined by:

$$r = \frac{r_0}{\beta} \quad -(23)$$

so the time dependence of β in this case is:

$$\beta(t) = -\frac{m r_0^2}{L t} \quad -(24)$$

and that of r is:

$$r(t) = -\frac{L}{m r_0}t. \quad -(25)$$

From Eqs. (23) and (10) r can be expressed in terms of ϕ :

$$r = \frac{r_0}{\tan^{-1}\left(\frac{L}{L_Z}\tan\phi\right)} \quad -(26)$$

and from Eqs. (23) and (11) r can be expressed in terms of θ :

$$r = \frac{r_0}{\sin^{-1}\left(\left(1-\left(\frac{L_Z}{L}\right)^2\right)^{-1/2}\cos\theta\right)} \quad -(27)$$

In general therefore r is a three dimensional function of ϕ and θ :

$$r = \frac{1}{2}r_0\left[\frac{1}{\tan^{-1}\left(\frac{L}{L_Z}\right)\tan\phi} + \frac{1}{\sin^{-1}\left(\left(1-\left(\frac{L_Z}{L}\right)^2\right)^{-1/2}\cos\theta\right)}\right] \quad -(28)$$

For an inverse square law of attraction:

$$F(r) = -\frac{mMG}{r^2} \qquad -(29)$$

where G is Newton's constant, the resulting three dimensional orbit is the beta conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\beta} \qquad -(30)$$

where α is the half right magnitude and ϵ the eccentricity. The dependence of β on t is given by:

$$t = \frac{m\alpha^2}{L}\int\frac{d\beta}{(1+\epsilon\cos\beta)^2} \qquad -(31)$$

where the time to complete one orbit is:

$$\tau = 2\pi\left(\frac{m\alpha^2}{L}\right) \qquad -(32)$$

Therefore:

$$t = \frac{\tau}{2\pi}\int_0^{\beta}\frac{d\beta}{(1+\epsilon\cos\beta)^2}$$

$$= \frac{\tau}{2\pi}\left[2\tan^{-1}\left(\left(\frac{1-\epsilon}{1+\epsilon}\right)^{1/2}\tan\frac{\beta}{2}\right) - \frac{\epsilon(1-\epsilon^2)^{1/2}\sin\beta}{1+\epsilon\cos\beta}\right] \qquad -(33)$$

For small ellipticity ϵ this can be inverted to give:

$$\beta(t) = 2\pi\frac{t}{\tau} + 2\epsilon\sin\left(2\pi\frac{t}{\tau}\right)$$

$$+\frac{5}{4}\epsilon^2\sin\left(4\pi\frac{t}{\tau}\right) + \frac{\epsilon^3}{12}\left(13\sin\left(6\pi\frac{t}{\tau}\right) - 3\sin\left(2\pi\frac{t}{\tau}\right)\right)$$

$$-(34)$$

$$+\;\ldots\ldots$$

However, the usual method used is Kepler's construction {11}, which gives:

$$t = \frac{\tau}{2\pi}\left(\phi - \epsilon \sin\phi\right) \quad -(35)$$

where:

$$\tan\frac{\beta}{2} = \left(\frac{1+\epsilon}{1-\epsilon}\right)^{1/2} \tan\frac{\phi}{2}. \quad -(36)$$

The quantity $2\pi t/\tau$ is the mean anomaly. Having found $\beta(t)$, Eqs. (10) and (11) may be used to find $\phi(t)$ and $\theta(t)$, and finally Eq. (30) may be used to find the three dimensional r (t) = f (theta(t), phi(t)). For the circle:

$$\epsilon = 0, \quad -(37)$$

$$r = \alpha, \quad -(38)$$

so:

$$\beta(t) = 2\pi\frac{t}{\tau} \quad -(39)$$

so the time dependence of ϕ from Eq. (10) reduces to:

$$\phi(t) = \tan^{-1}\left(\frac{L_Z}{L}\tan\left(\frac{Lt}{m\alpha^2}\right)\right). \quad -(40)$$

In this case the time dependencies of $\phi(t)$ and $\theta(t)$ are analytical:

$$\phi(t) = \tan^{-1}\left(\frac{L_Z}{L}\tan\left(2\pi\frac{t}{\tau}\right)\right) \quad -(41)$$

and

$$\theta(t) = \cos^{-1}\left(\left(1-\left(\frac{L_Z}{L}\right)^2\right)^{1/2}\sin\left(2\pi\frac{t}{\tau}\right)\right) \quad -(42)$$

The overall time dependence of r is found from the conservation of angular momentum:

$$\frac{d\beta}{dt} = \frac{L}{mr^2} \quad -(43)$$

and the time dependence of β .

The polar equations considered above are supplemented by a Cartesian analysis and classification as follows, a classification which allows direct comparison with results from solid geometry, a part of Cartan geometry upon which ECE theory is based. The Cartesian representations all emerge from the beta conic section:

$$r = \frac{\alpha}{1 + \epsilon \cos \beta} \quad -(44)$$

in polar representation. This conic section is equivalent to the hamiltonian ($|$) and the lagrangian (∂) with an inverse squ are force law of attraction:

$$F(r) = -mMG / r^2 \quad -(45)$$

1) Beta Ellipse

The Cartesian representation of the beta ellipse with an attracting mass M at one focus of the ellipse is:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad -(46)$$

and

$$Z^2 = \left(1 - \frac{L_Z}{L}\right)^2 Y^2 \quad -(47)$$

with

$$Z = -\left(1 - \frac{L_Z}{L}\right) Y \quad -(48)$$

Here:

$$X = a\epsilon + r\cos\beta \quad -(49)$$
$$Y = r\sin\beta \quad -(50)$$
$$Z = r\cos\theta \quad -(51)$$

where a and b are the major and minor semi axes. The ellipticity of the ellipse is:

$$0 < \epsilon^2 = \left(1 - \frac{b^2}{a^2}\right) < 1 \quad -(52)$$

and its half right latitude or semi latus rectum is:

$$d = a\left(1 - \epsilon^2\right). \quad -(53)$$

2) Beta Hyperbola

In this case:

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1, \quad -(54)$$
$$Z^2 = \left(1 - \frac{Lz}{L}\right)^2 Y^2 \quad -(55)$$

where:

$$X = -a\epsilon + r\cos\beta \quad -(56)$$
$$Y = r\sin\beta \quad -(57)$$
$$Z = r\cos\theta \quad -(58)$$

The ellipticity is:

$$\epsilon^2 = 1 + \frac{b^2}{a^2} > 1 \quad -(59)$$

and the half right latitude is:

$$d = a\left(\epsilon^2 - 1\right). \quad -(60)$$

3) Beta Parabola

Here:

$$y^2 = 4aX \quad -(61)$$

$$z^2 = \left(1 - \frac{L_z}{L}\right)^2 y^2 \quad -(62)$$

and the ellipticity is unity:

$$\epsilon = 1 \quad -(63)$$

4) Beta Circle

Here:

$$x^2 + y^2 = r^2 \quad -(64)$$

$$z^2 = \left(1 - \frac{L_z}{L}\right)^2 y^2 \quad -(65)$$

and the ellipticity is zero:

$$\epsilon = 0 \quad -(66)$$

$$r = d = a = b \quad -(67)$$

$$X = r\cos\beta \quad -(68)$$

$$Y = r\sin\beta \quad -(69)$$

$$Z = r\cos\theta \quad -(70)$$

Using these equations a Cartesian classification can be made of three dimensional orbits into sixteen fundamental types as follows.

Ellipse

Type (1) : Ellipsoidal

$$\frac{x^2}{a^2} + y^2\left(\frac{1}{b^2} - \frac{1}{c^2}\left(1 - \frac{L_z}{L}\right)\right) + \frac{z^2}{c^2} = 1 \quad -(71)$$

Type (2): One Sheet Hyperbeloidal

$$\frac{x^2}{a^2} + y^2\left(\frac{1}{b^2} + \frac{1}{c^2}\left(1-\frac{L_z}{L}\right)\right) - \frac{z^2}{c^2} = 1 \quad -(72)$$

Type (3): Elliptic Parabaloidal

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z}{c} = 1 + \frac{y}{c}\left(1-\frac{L_z}{L}\right) \quad -(73)$$

Type (4): Elliptic Parabaloidal

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c} = 1 - \frac{y}{c}\left(1-\frac{L_z}{L}\right) \quad -(74)$$

Hyperbola

Type (5): One Sheet Hyperbeloidal

$$\frac{x^2}{a^2} - y^2\left(\frac{1}{b^2} + \frac{1}{c^2}\left(1-\frac{L_z}{L}\right)\right) + \frac{z^2}{c^2} = 1 \quad -(75)$$

Type (6): Two Sheet Hyperbeloidal

$$\frac{x^2}{a^2} - y^2\left(\frac{1}{b^2} - \frac{1}{c^2}\left(1-\frac{L_z}{L}\right)\right) - \frac{z^2}{c^2} = 1 \quad -(76)$$

Type (7): Hyperbolic Parabaloidal

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z}{c} = 1 - \left(1-\frac{L_z}{L}\right)\frac{y}{c} \quad -(77)$$

Type (8): Hyperbolic Parabaloidal

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z}{c} = 1 + \left(1 - \frac{L_z}{L}\right)\frac{y}{c} \quad -(78)$$

Parabola

Type (9): Elliptic Parabaloidal

$$y^2\left(\frac{1}{a^2} - \frac{1}{b^2}\left(1 - \frac{L_z}{L}\right)\right) + \frac{z^2}{b^2} = \frac{4x}{a} \quad -(79)$$

Type (10): Hyperbolic Parabaloidal

$$y^2\left(\frac{1}{a^2} + \frac{1}{b^2}\left(1 - \frac{L_z}{L}\right)\right) - \frac{z^2}{b^2} = \frac{4x}{a} \quad -(80)$$

Type (11): Parabaloid

$$\frac{y^2}{a^2} + \frac{z}{b} = \frac{4x}{a} - \frac{1}{b}\left(1 - \frac{L_z}{L}\right)y \quad -(81)$$

Type (12): Parabaloid

$$\frac{y^2}{a^2} - \frac{z}{b} = \frac{4x}{a} + \frac{1}{b}\left(1 - \frac{L_z}{L}\right)y \quad -(82)$$

Circle

Type (13): Ellipsoidal

$$x^2 + y^2\left(1 - \left(1 - \frac{L_z}{L}\right)^2\right) + z^2 = r^2 \quad -(83)$$

Type (14): One Sheet Hyperbeloidal

$$X^2 + Y^2\left(1 + \left(1 - \frac{Lz}{L}\right)^2\right) - Z^2 = r^2 \quad -(84)$$

Type (15): Ellipsoidal Parabolic

$$X^2 + Y^2 + aZ = r^2 - aY\left(1 - \frac{Lz}{L}\right) \quad -(85)$$

Type (16): Ellipsoidal Parabolic

$$X^2 + Y^2 - aZ = r^2 + aY\left(1 - \frac{Lz}{L}\right) \quad -(86)$$

Types (1) to (4) are given by the beta ellipse with eccentricity:

$$0 < \epsilon < 1. \quad -(87)$$

Types (5) to (8) are given by the beta hyperbola with eccentricity:

$$\epsilon > 1. \quad -(88)$$

Types (9) to (12) are given by the beta parabola with eccentricity:

$$\epsilon = 1. \quad -(89)$$

Types (13) to (16) are given by the beta circle with eccentricity:

$$\epsilon = 0. \quad -(90)$$

3. GRAPHICAL ANALYSIS

Section by Dr. Horst Eckardt

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