

□ 1 Equations

```
(%i1) kill(all);
(%o0) done
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```
(%i1) E3: d2 = (x^2-1)*L^2/(m*r^3)-x^2*L^2/(alpha*m*r^2)+L^2/(m*r^3);
(%o1) d2 = -\frac{x^2 L^2}{\alpha m r^2} + \frac{(x^2-1)L^2}{m r^3} + \frac{L^2}{m r^3}
```

```
(%i2) E3a: ratsimp(E3);
(%o2) d2 = -\frac{(r-\alpha)x^2 L^2}{\alpha m r^3}
```

```
(%i3) E3b: expand(ratsubst(L^2/(m^2*M*G), alpha, E3a));
(%o3) d2 = \frac{x^2 L^2}{m r^3} - \frac{m x^2 G M}{r^2}
```

```
(%i4) E11: m*d2r = -m*M*G/r^2+L^2/(m*r^3)-3*G*M*L^2/(m*c^2*r^4);
(%o4) d2r m = -\frac{3 G L^2 M}{c^2 m r^4} - \frac{m G M}{r^2} + \frac{L^2}{m r^3}
```

□ 2 Comparison of Eqs.(3),(11)

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(%i5) E12: rhs(E3b) = rhs(E11);
(%o5) \frac{x^2 L^2}{m r^3} - \frac{m x^2 G M}{r^2} = -\frac{3 G L^2 M}{c^2 m r^4} - \frac{m G M}{r^2} + \frac{L^2}{m r^3}
```

```
(%i6) E12a: first(solve(E12, x^2));
(%o6) x^2 = \frac{(3 G L^2 + c^2 m^2 r^2 G) M - c^2 r L^2}{c^2 m^2 r^2 G M - c^2 r L^2}
```

```
(%i7) A: -m*M*G/r^2+L^2/(m*r^3);
(%o7) \frac{L^2}{m r^3} - \frac{m G M}{r^2}
```

```
(%i8) E14: x2 = 1-3*G*M*L^2/(A*m*c^2*r^4);
(%o8) x2 = 1 - \frac{3 G L^2 M}{c^2 m r^4 \left( \frac{L^2}{m r^3} - \frac{m G M}{r^2} \right)}
```

```
(%i9) ratsimp(E12a-E14);
(%o9) x^2 - x2 = 0
```

□ 3 Plots

```

(%i10) kill(x);
(%o10) done

(%i11) E16: xx: 1+3*M*G/(c^2*alpha);
(%o11)  $\frac{3GM}{\alpha c^2} + 1$ 

(%i12) xx: expand(ratsubst(L^2/(m^2*alpha), G*M, xx));
(%o12)  $\frac{3L^2}{\alpha^2 c^2 m^2} + 1$ 

(%i13) E17: xE2a: (expand(ratsimp(rhs(E14))));
(%o13)  $\frac{3GL^2M}{c^2m^2r^2GM-c^2rL^2} + \frac{c^2m^2r^2GM}{c^2m^2r^2GM-c^2rL^2} - \frac{c^2rL^2}{c^2m^2r^2GM-c^2rL^2}$ 

(%i14) xE2b: factor(ratsubst(L^2/(m^2*alpha), G*M, xE2a));
(%o14)  $\frac{3L^2+c^2m^2r^2-\alpha c^2m^2r}{c^2m^2r(r-\alpha)}$ 

(%i15) xE2: (3*L^2)/(c^2*m^2*r*(r-alpha))+1;
(%o15)  $\frac{3L^2}{c^2m^2r(r-\alpha)} + 1$ 

(%i16) ratsimp(xE2-xE2b);
(%o16) 0

(%i17) xE: sqrt(xE2);
(%o17)  $\sqrt{\frac{3L^2}{c^2m^2r(r-\alpha)} + 1}$ 

(%i18) theta: 1/x*acos(1/epsilon*(alpha/r-1));
(%o18)  $\frac{\operatorname{acos}\left(\frac{\frac{\alpha}{r}-1}{\epsilon}\right)}{x}$ 

(%i19) theta1: ev(theta, [x=xx]);
(%o19)  $\frac{\operatorname{acos}\left(\frac{\frac{\alpha}{r}-1}{\epsilon}\right)}{\frac{3L^2}{\alpha^2 c^2 m^2} + 1}$ 

```

```
(%i20) theta2: ev(theta, [x=xE]);
```

$$(\%o20) \frac{\arccos\left(\frac{\frac{\alpha}{r}-1}{\varepsilon}\right)}{\sqrt{\frac{3L^2}{c^2 m^2 r(r-\alpha)}+1}}$$

```
(%i21) th1: ev(theta1, [alpha=1, epsilon=0.3, L=1, m=1, c=10]);
```

$$(\%o21) \frac{100 \arccos\left(3.333333333333334\left(\frac{1}{r}-1\right)\right)}{103}$$

```
(%i22) th2: ev(theta2, [alpha=1, epsilon=0.3, L=1, m=1, c=10]);
```

$$(\%o22) \frac{\arccos\left(3.333333333333334\left(\frac{1}{r}-1\right)\right)}{\sqrt{\frac{3}{100(r-1)r}+1}}$$

```
(%i23) wxplot2d([th1,th2], [r,0,1.5], [y,0,5],
  [ylabel, "theta"], [legend, "x from x theory", "x from Einstein theory"]);
```

plot2d: expression evaluates to non-numeric value somewhere in plotting range

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plot2d: some values were clipped.

```
(%t23)
```

