1 Definition of curl and div in cylindrical coordi

```
(%i1) kill(all); numer: false;
(%o0) done
(%o1) false
```

Curl op.

```
(%i2) curl(v) := [1/r*diff(v[3],theta)-diff(v[2],z),
                          diff(v[1],z)-diff(v[3],r),
                          1/r*(diff(r*v[2],r)-diff(v[1],theta))];
```

(%o2) $\mathrm{curl}(v):=[\frac{1}{r}\mathrm{diff}(v_3,\theta)-\mathrm{diff}(v_2,z),\mathrm{diff}(v_1,z)-\mathrm{diff}(v_3,r),\frac{1}{r}(\mathrm{diff}(r\,v_2,r)-\mathrm{diff}(v_1,\theta))]$

Div op.

```
(%i3) div(v) := 1/r*diff(r*v[1],r) + 1/r*diff(v[2],theta) + diff(v[3],z)
```

(%o3) $\mathrm{div}(v):=\frac{1}{r}\mathrm{diff}(r\,v_1,r)+\frac{1}{r}\mathrm{diff}(v_2,\theta)+\mathrm{diff}(v_3,z)$

2 Coordinate transformations

Coordinate Transf. cyl. --> cartesian

```
(%i4) Tc(x) := [x[1]*cos(x[2]),
                x[1]*sin(x[2]),
              x[3]];
```

(%o4) $\mathrm{Tc}(x):=[x_1\cos(x_2),x_1\sin(x_2),x_3]$

Vector Transf. cyl. --> cartesian

```
(%i5) T(w) := [w[1]*cos(theta) - w[2]*sin(theta),
               w[1]*sin(theta) + w[2]*cos(theta),
              w[3]];
```

(%o5) $\mathrm{T}(w):=[w_1\cos(\theta)-w_2\sin(\theta),w_1\sin(\theta)+w_2\cos(\theta),w_3]$

3 vector field v (no radial component, see Marsh)

```
(%i6) depends(B,r);
```

(%o6) $[\mathrm{B}(r)]$

```
(%i7) v: [0, B[theta], B[z]];
```

(%o7) $[0,B_\theta,B_z]$

Curl of v

```
(%i8) cv: ratsimp(curl(v));
```

(%o8) $$\left[0, -\frac{d}{d\,r}B_z, \frac{r\left(\frac{d}{d\,r}B_\theta\right)+B_\theta}{r}\right]$$

Check of Beltrami condition, per component

```
(%i9) a2: cv[2]/v[2];
```

(%o9) $$-\frac{\frac{d}{d\,r}B_z}{B_\theta}$$

```
(%i10) a3: cv[3]/v[3];
```

(%o10) $$\frac{r\left(\frac{d}{d\,r}B_\theta\right)+B_\theta}{r\,B_z}$$

Condition for equal Beltrami scalar function of all components

```
(%i11) E1: a2=a3;
```

(%o11) $$-\frac{\frac{d}{d\,r}B_z}{B_\theta}=\frac{r\left(\frac{d}{d\,r}B_\theta\right)+B_\theta}{r\,B_z}$$

```
(%i12) E2: E1*B[theta]*r*B[z];
```

(%o12) $$-r\,B_z\left(\frac{d}{d\,r}B_z\right)=B_\theta\left(r\left(\frac{d}{d\,r}B_\theta\right)+B_\theta\right)$$

```
(%i13) ode2(E2, B[z], r);
```

(%o13) $$-\frac{B_z^2}{2\,B_\theta}=(\log(r)+1)B_\theta+\%c$$

```
(%i14) ode2(E2, B[theta], r);
```

(%o14) $$-\frac{2\,B_z\int r^2\left(\frac{d}{d\,r}B_z\right)d\,r+r^2\,B_\theta^2}{2}=\%c$$

div of v

```
(%i15) div(v);
```

(%o15) 0

4 vector field v (Bessel functions)

```
(%i16) kill(B);
```
(%o16) *done*

```
(%i17) v: B[0]*[0, bessel_j(1, kappa*r), bessel_j(0, kappa*r)];
```
(%o17) $[0, B_0\,\mathrm{bessel\_j}(1,\kappa\,r), B_0\,\mathrm{bessel\_j}(0,\kappa\,r)]$

Curl of v

```
(%i18) cv: ratsimp(curl(v));
```
(%o18) $[0, B_0\,\mathrm{bessel\_j}(1,\kappa\,r)\,\kappa, -\dfrac{(B_0\,\mathrm{bessel\_j}(2,\kappa\,r) - B_0\,\mathrm{bessel\_j}(0,\kappa\,r))\,\kappa\,r - 2\,B_0\,\mathrm{bessel\_j}(1,\kappa\,r)}{2\,r}]$

Check of Beltrami condition, per component

```
(%i19) cv[2]/v[2];
```
(%o19) κ

```
(%i20) ratsimp(cv[3]/v[3]);
```
(%o20) $-\dfrac{(\mathrm{bessel\_j}(2,\kappa\,r) - \mathrm{bessel\_j}(0,\kappa\,r))\,\kappa\,r - 2\,\mathrm{bessel\_j}(1,\kappa\,r)}{2\,\mathrm{bessel\_j}(0,\kappa\,r)\,r}$

div of v

```
(%i21) div(v);
```
(%o21) 0

5 vector field v (Lundquist solution functions)

```
(%i22) v: [%lambda*exp(-%lambda*z)*bessel_j(1, sqrt(alpha^2+%lambda^2)*r)
        alpha* exp(-%lambda*z)*bessel_j(1, sqrt(alpha^2+%lambda^2)*r),
        sqrt(alpha^2+%lambda^2)*exp(-%lambda*z)*bessel_j(0, sqrt(alpha
```
(%o22) $[\mathrm{bessel\_j}(1,\sqrt{\alpha^2+\lambda^2}\,r)\,\lambda\,\%e^{-\lambda\,z}, \mathrm{bessel\_j}(1,\sqrt{\alpha^2+\lambda^2}\,r)\,\alpha\,\%e^{-\lambda\,z}, \mathrm{bessel\_j}(0,\sqrt{\alpha^2+\lambda^2}\,r)\sqrt{\alpha^2+\lambda^2}\,\%e^{-\lambda\,z}]$

```
(%i23) v: ratsubst(kappa, sqrt(alpha^2+%lambda^2), v);
```
(%o23) $[\mathrm{bessel\_j}(1,\kappa\,r)\,\lambda\,\%e^{-\lambda\,z}, \mathrm{bessel\_j}(1,\kappa\,r)\,\alpha\,\%e^{-\lambda\,z}, \mathrm{bessel\_j}(0,\kappa\,r)\,\kappa\,\%e^{-\lambda\,z}]$

Curl of v

```
(%i24) cv: ratsimp(curl(v));
```
(%o24) $[\mathrm{bessel\_j}(1,\kappa r)\lambda\alpha \%e^{-\lambda z}, (\mathrm{bessel\_j}(1,\kappa r)\kappa^2-\mathrm{bessel\_j}(1,\kappa r)\lambda^2)\%e^{-\lambda z}, -\frac{((\mathrm{bessel\_j}(2,\kappa r)-\mathrm{bessel\_j}(0,\kappa r))\alpha\kappa r-2\,\mathrm{bessel\_j}(1,\kappa r)\alpha)\%e^{-\lambda z}}{2r}]$

```
(%i25) kappa: sqrt(alpha^2+%lambda^2);
```
(%o25) $\sqrt{\alpha^2+\lambda^2}$

div of v

```
(%i26) ratsimp(div(v));
```
(%o26) $-\frac{((\mathrm{bessel\_j}(2,\kappa r)+\mathrm{bessel\_j}(0,\kappa r))\lambda\kappa r-2\,\mathrm{bessel\_j}(1,\kappa r)\lambda)\%e^{-\lambda z}}{2r}$

Check of Check of Beltrami condition, per component

```
(%i27) cv[1]/v[1];
```
(%o27) α

```
(%i28) ratsimp(ev(cv[2]/v[2]));
```
(%o28) α

```
(%i29) ratsimp(cv[3]/v[3]);
```
(%o29) $-\frac{(\mathrm{bessel\_j}(2,\kappa r)-\mathrm{bessel\_j}(0,\kappa r))\alpha\kappa r-2\,\mathrm{bessel\_j}(1,\kappa r)\alpha}{2\,\mathrm{bessel\_j}(0,\kappa r)\kappa r}$

```
(%i30) ratsimp(ev(cv[3]/v[3]));
```
(%o30) $-(\left(\mathrm{bessel\_j}(2,\sqrt{\alpha^2+\lambda^2}\,r)-\mathrm{bessel\_j}(0,\sqrt{\alpha^2+\lambda^2}\,r)\right)\alpha\sqrt{\alpha^2+\lambda^2}\,r-2\,\mathrm{bessel\_j}(1,\sqrt{\alpha^2+\lambda^2}\,r)\alpha)/(2\,\mathrm{bessel\_j}(0,\sqrt{\alpha^2+\lambda^2}\,r)\sqrt{\alpha^2+\lambda^2}\,r)$

6 Plot data for first layer, X-Y and Z components

```
(%i31) /*filebase: "D:/Doc/Artikel-Eck/ECE-Theorie/Paper257/"*/
       filebase: "F:/Paper258/";
(%o31) F:/Paper258/
```

```
(%i32) filename: concat(filebase, "x2.dat");
(%o32) F:/Paper258/x2.dat
```

```
(%i33) numer: true;
       stream: openw(filename);
       printf(stream, "# x1 x2 x3 v1 v2 v3 cv1 cv2 cv3~%");
(%o33) true
(%o34) Stream [STRING-CHAR]
(%o35) false

(%i36) alpha: 1$
       %lambda: 1$
       kappa: ev(kappa);
(%o38) 1.414213562373095

(%i39) x: [r, theta, z];
(%o39) [r,θ,z]

(%i40) Tx: Tc(x);
(%o40) [r cos(θ), r sin(θ), z]

(%i41) Tv: T(v)$

(%i42) Tcv: T(cv)$

(%i43) for k:0 thru 2 do (
         nf: 0,
         z: k,
         for th:0 thru 7 do (
           theta: 0.001+ th*%pi/4,
           for i:0 thru 11 do (
               r: 0.0001+i/2.,
               x1: ev(Tx),
               v1: ev(Tv),
               cv1: ev(Tcv),
       /*print (x1,v1,cv1),*/
               w1: x1,
               w2: v1,
               w3: cv1,
               nf: nf+1,
               wa[nf]: append(w1, w2, w3)
       )),
       for n:1 thru nf do write_data(wa[n], stream),
           printf(stream, "~%"),
           printf(stream, "~%")
       );
(%o43) done

(%i44) close(stream);
(%o44) true
```