1 Definition of curl in cylindrical coordinates

```
(%i1) curl_r(vr,vth,vz) := 1/r*diff(vz,theta)-diff(vth,z);
```

$$(\%o1)\quad \mathrm{curl\_r}(vr,vth,vz):=\frac{1}{r}\,\mathrm{diff}(vz,\theta)-\mathrm{diff}(vth,z)$$

```
(%i2) curl_th(vr,vth,vz) := diff(vr,z)-diff(vz,r);
```

$$(\%o2)\quad \mathrm{curl\_th}(vr,vth,vz):=\mathrm{diff}(vr,z)-\mathrm{diff}(vz,r)$$

```
(%i3) curl_z(vr,vth,vz) := 1/r*(diff(r*vth,r)-diff(vr,theta));
```

$$(\%o3)\quad \mathrm{curl\_z}(vr,vth,vz):=\frac{1}{r}\left(\mathrm{diff}(r\,vth,r)-\mathrm{diff}(vr,\theta)\right)$$

2 Examples

2.1 linear

```
(%i4) vr: 0;
```

$$(\%o4)\quad 0$$

```
(%i5) vth: (r/r0)*z0;
```

$$(\%o5)\quad \frac{r\,z0}{r0}$$

```
(%i6) vz: ((r0-r)/r0)*z0;
```

$$(\%o6)\quad \frac{(r0-r)\,z0}{r0}$$

```
(%i7) cvr: [curl_r(vr,vth,vz), curl_th(vr,vth,vz), curl_z(vr,vth,vz)];
```

$$(\%o7)\quad \left[0,\frac{z0}{r0},\frac{2\,z0}{r0}\right]$$

2.2 quadratic

```
(%i8) vr: 0;
```

$$(\%o8)\quad 0$$

```
(%i9) vth: (r/r0)^2*z0;
```

$$(\%o9)\quad \frac{r^2\,z0}{r0^2}$$

```
(%i10) vz: ((r0-r)/r0)^2*z0;
```

$$(\%o10)\quad \frac{(r0-r)^2\,z0}{r0^2}$$

```
(%i11) cvr: [curl_r(vr,vth,vz), curl_th(vr,vth,vz), curl_z(vr,vth,vz)];
```

$$(\%o11)\quad \left[0,\frac{2(r0-r)z0}{r0^2},\frac{3\,r\,z0}{r0^2}\right]$$

2.3 square root

```
(%i12) vr: 0;
```

$$(\%o12)\quad 0$$

```
(%i13) vth: sqrt(r/r0)*z0;
```

$$(\%o13)\quad \sqrt{\frac{r}{r0}}\,z0$$

```
(%i14) vz: sqrt((r0-r)/r0)*z0;
```

$$(\%o14)\quad \sqrt{\frac{r0-r}{r0}}\,z0$$

```
(%i15) cvr: [curl_r(vr,vth,vz), curl_th(vr,vth,vz), curl_z(vr,vth,vz)];
```

$$(\%o15)\quad \left[0,\frac{z0}{2\,r0\sqrt{\frac{r0-r}{r0}}},\frac{\frac{r\,z0}{2\sqrt{\frac{r}{r0}}\,r0}+\sqrt{\frac{r}{r0}}\,z0}{r}\right]$$

2.4 z rotation

```
(%i16) vr: 0;
```

$$(\%o16)\quad 0$$

```
(%i17) vth: (r/r0)*z0;
```

$$(\%o17)\quad \frac{r\,z0}{r0}$$

```
(%i18) vz: ((r0-r)/r0)*z0*cos(kappa*z);
```

$$(\%o18)\quad \frac{(r0-r)\cos(\kappa\,z)z0}{r0}$$

```
(%i19) cvr: [curl_r(vr,vth,vz), curl_th(vr,vth,vz), curl_z(vr,vth,vz)];
```

$$(\%o19)\quad \left[0,\frac{\cos(\kappa\,z)z0}{r0},\frac{2\,z0}{r0}\right]$$

2.5 plane wave

```
(%i20) vr: r0;
```

$$(\%o20)\quad r0$$

```
(%i21) vth: cos(kappa*z);
```
$$(\%o21)\quad \cos(\kappa\, z)$$

```
(%i22) vz: 0;
```
$$(\%o22)\quad 0$$

```
(%i23) cvr: [curl_r(vr,vth,vz), curl_th(vr,vth,vz), curl_z(vr,vth,vz)];
```
$$(\%o23)\quad [\kappa \sin(\kappa\, z), 0, \frac{\cos(\kappa\, z)}{r}]$$

```
(%i24) vr: 0;
```
$$(\%o24)\quad 0$$

```
(%i25) vth: (r/r0)^-2*z0;
```
$$(\%o25)\quad \frac{r0^2\, z0}{r^2}$$

```
(%i26) vz: ((r0-r)/r0)^-2*z0;
```
$$(\%o26)\quad \frac{r0^2\, z0}{(r0-r)^2}$$

```
(%i27) cvr: [curl_r(vr,vth,vz), curl_th(vr,vth,vz), curl_z(vr,vth,vz)];
```
$$(\%o27)\quad [0, -\frac{2\, r0^2\, z0}{(r0-r)^3}, -\frac{r0^2\, z0}{r^3}]$$

3 Cartesian

```
(%i28) curl(a) := [diff(a[3],y) - diff(a[2],z),
                   diff(a[1],z) - diff(a[3],x),
                   diff(a[2],x) - diff(a[1],y)];
```
$$(\%o28)\quad \mathrm{curl}(a) := [\mathrm{diff}(a_3, y) - \mathrm{diff}(a_2, z), \mathrm{diff}(a_1, z) - \mathrm{diff}(a_3, x), \mathrm{diff}(a_2, x) - \mathrm{diff}(a_1, y)]$$

```
(%i29) v: [r*cos(kappa*z),r*sin(kappa*z),0];
```
$$(\%o29)\quad [r \cos(\kappa\, z), r \sin(\kappa\, z), 0]$$

```
(%i30) curl(v);
```
$$(\%o30)\quad [-\kappa\, r \cos(\kappa\, z), -\kappa\, r \sin(\kappa\, z), 0]$$

```
(%i31) v: [-o*y,o*x,a(x,y)];
```
$$(\%o31)\quad [-o\, y, o\, x, a(x, y)]$$

```
(%i32) curl(v);
```
$$(\%o32)\quad [\frac{d}{d\, y} a(x, y), -\frac{d}{d\, x} a(x, y), 2\, o]$$

4 General solution

```
(%i33) depends([TX,TY,TZ],[x,y,z]);
(%o33) [TX(x,y,z),TY(x,y,z),TZ(x,y,z)]
```

```
(%i90)                    BX:  T2* sin( T*z) + T3* cos( T*y);
                          BY:  T3* sin( T*x) + T1* cos( T*z);
                          BZ:  T1* sin( T*y) + T2* cos( T*x);
(%o90) cos(y T)T3+sin(z T)T2
(%o91) sin(x T)T3+cos(z T)T1
(%o92) cos(x T)T2+sin(y T)T1
```

```
(%i93) curl([BX,BY,BZ]);
(%o93) [T sin(z T)T1+T cos(y T)T1,T cos(z T)T2+T sin(x T)T2,T sin(y T)T3+
T cos(x T)T3]
```

```
(%i94) factor(ratsimp(%));
(%o94) [T(sin(z T)+cos(y T))T1,T(cos(z T)+sin(x T))T2,T
(sin(y T)+cos(x T))T3]
```

```
(%i98)                BX:  T2* sin( T*z) + T3* cos(omega*t-kappa*z);
                      BY:  T3* sin( T*x) + T1* cos(omega*t-kappa*z);
                      BZ:  T1* sin( T*y) + T2* cos( T*x);
(%o98) cos(κ z-ω t)T3+sin(z T)T2
(%o99) sin(x T)T3+cos(κ z-ω t)T1
(%o100) cos(x T)T2+sin(y T)T1
```

```
(%i101) curl([BX,BY,BZ]);
(%o101) [T cos(y T)T1+κ sin(κ z-ω t)T1,-κ sin(κ z-ω t)T3+T cos(z T)T2+T
sin(x T)T2,T cos(x T)T3]
```

```
(%i102) factor(ratsimp(%));
(%o102) [(T cos(y T)+κ sin(κ z-ω t))T1,-
(κ sin(κ z-ω t)T3-T cos(z T)T2-T sin(x T)T2),T cos(x T)T3]
```

5 Generating B from a potential function

```
(%i124) depends([phi],[x,y,t]);
(%o124) [ϕ(x,y,t)]
```

```
(%i125) depends([w],[x,y,z,t]);
(%o125) [w(x,y,z,t)]
```

```
(%i126) BX: -diff(phi,y);
        BY: diff(phi,x);
        BZ: w;
```

$$(\%o126)\quad -\frac{d}{d\,y}\phi$$

$$(\%o127)\quad \frac{d}{d\,x}\phi$$

$$(\%o128)\quad w$$

```
(%i137) c: curl([BX,BY,BZ]);
```

$$(\%o137)\quad \left[\frac{d}{d\,y}w,-\frac{d}{d\,x}w,\frac{d^2}{d\,y^2}\phi+\frac{d^2}{d\,x^2}\phi\right]$$

Beltrami conditions

```
(%i141) c[1]=BX;
        c[2]=BY;
        c[3]=BZ;
```

$$(\%o141)\quad \frac{d}{d\,y}w=-\frac{d}{d\,y}\phi$$

$$(\%o142)\quad -\frac{d}{d\,x}w=\frac{d}{d\,x}\phi$$

$$(\%o143)\quad \frac{d^2}{d\,y^2}\phi+\frac{d^2}{d\,x^2}\phi=w$$

5.1 Example

```
(%i197) phi: (cos(x)+sin(y))*cos(t);
```

$$(\%o197)\quad \cos(t)\,(\sin(y)+\cos(x))$$

```
(%i198) w: -phi;
```

$$(\%o198)\quad -\cos(t)\,(\sin(y)+\cos(x))$$

```
(%i199) B: ev([BX,BY,BZ],diff);
```

$$(\%o199)\quad [-\cos(t)\cos(y),-\cos(t)\sin(x),-\cos(t)\,(\sin(y)+\cos(x))]$$

curl(B)=B

```
(%i200) ev(curl([BX,BY,BZ]),diff);
```

$$(\%o200)\quad [-\cos(t)\cos(y),-\cos(t)\sin(x),-\cos(t)\sin(y)-\cos(t)\cos(x)]$$

```
(%i201) factor(%);
```

$$(\%o201)\quad [-\cos(t)\cos(y),-\cos(t)\sin(x),-\cos(t)\,(\sin(y)+\cos(x))]$$