1) 255(3): Proof that Zero Torsion means Zero Curvature

Consider the vector format of the Cartan identity derived in UFT 254:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a + \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b := \underline{q}^b \cdot \underline{R}^a{}_b \quad -(1)$$

If torsion is neglected:

$$\underset{\sim}{T}^a = ?\ \underline{0} \quad -(2)$$

then

$$\underline{q}^b \cdot \underline{R}^a{}_b = 0 \quad -(3)$$

A possible solution of eq. (3) is:

$$\underline{R}^a{}_b = \underline{0} \quad -(4)$$

So if torsion is zero, curvature is zero, QED.

This conclusion is alone enough to refute the entire era of Einsteinian general relativity.

If it is argued that:

$$\underline{q}^b \cdot \underline{R}^a{}_b = 0 \quad -(5)$$

then in tensor notation this is equivalent to the obsolete first Bianchi identity:

$$R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} = 0 \quad -(6)$$

Eq. (6) means that in the obsolete Einsteinian era:

$$\underline{q}^b \cdot \underline{R}^a{}_b = 0, \quad -(7)$$

$$\underline{R}^a{}_b \neq \underline{0} \quad -(8)$$

$$\underset{\sim}{T}^a = \underline{0} \quad -(9)$$

By definition eq. (7) means that:

$$\underline{q}^b \cdot \left( \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \right) = 0 \quad -(10)$$

Eq. (8) means that the only possible solution of eq. (10) compatible with eqs. (7) to (9) is:

$$\underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b = 0 \quad -(11)$$

$$\underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b = 0 \quad -(12)$$

From UFT254 it was found that the Cartan identity is the well known vector identity:

$$\underline{\nabla} \cdot \underline{\omega}^b{}_c \times \underline{q}^c = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b - \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(13)$$

So eq. (11) means that the Cartan identity is reduced to:

$$\underline{\nabla} \cdot \underline{\omega}^b{}_c \times \underline{q}^c = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b \quad -(14)$$

In arriving at eq. (14) it has been assumed that

$$\underset{\sim}{T}^b = \underline{\nabla} \times \underline{q}^b - \underline{\omega}^a{}_b \times \underline{q}^b = \underline{0} \quad -(15)$$

i.e. that:

$$\underline{\nabla} \times \underline{q}^b = \underline{\omega}^a{}_b \times \underline{q}^b \quad -(16)$$

From eq. (16) in eq. (14):

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q}^b = 0 \quad -(17)$$

and

$$\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{q}^c = 0 \quad -(18)$$

Eq. (18) is eq. (12) self consistently and eq. (17) is a vector identity, QED.

3) The obsolete first Bianchi identity is therefore either eq. (11) or eq. (12).

Note that a possible solution of eqs. (11) and (12) is

$$\underline{R}^a{}_b = \epsilon^a{}_{bc} \underset{\sim}{T}^c \quad -(19)$$

$$= \underline{0},$$

so it is again concluded that the curvature vanishes if the torsion vanishes.

In previous work the same conclusion was reached using the commutator of covariant derivatives (e.g. UFT 139)

In tensor notation eq. (11) is:

$$\partial_\mu \omega^a{}_{\nu b} - \partial_\nu \omega^a{}_{\mu b} = 0 \quad -(20)$$

and eq. (12) is:

$$\omega^a{}_{\mu c} \omega_\nu{}^c{}_b - \omega^a{}_{\nu c} \omega_\mu{}^c{}_b = 0 \quad -(21)$$

so

$$R^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{\nu b} - \partial_\nu \omega^a{}_{\mu b} + \omega^a{}_{\mu c} \omega_\nu{}^c{}_b - \omega^a{}_{\nu c} \omega_\mu{}^c{}_b$$

$$= 0 \quad -(22)$$

Q.E.D

The curvature again vanishes if torsion is zero.

The obsolete first Bianchi identity (6) is true if and only if:

4)

$$R^a{}_{\mu\nu\rho} = R^a{}_{\rho\mu\nu} = R^a{}_{\nu\rho\mu} = 0 \quad -(23)$$

The correct identities must always be the Cartan identity:

$$D_\mu T^a{}_{\nu\rho} + D_\rho T^a{}_{\mu\nu} + D_\nu T^a{}_{\rho\mu} := R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} \quad -(24)$$

and the Evans identity:

$$D_\mu \tilde{T}^a{}_{\nu\rho} + D_\rho \tilde{T}^a{}_{\mu\nu} + D_\nu \tilde{T}^a{}_{\rho\mu} := \tilde{R}^a{}_{\mu\nu\rho} + \tilde{R}^a{}_{\rho\mu\nu} + \tilde{R}^a{}_{\nu\rho\mu} \quad -(25)$$

These have been proven in all detail in the UFT papers.

The vector notation for eqs. (24) and (25) is:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a + \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b := \underline{v}^b \cdot \underline{R}^a{}_b \quad -(26)$$

and

$$\underline{\nabla} \cdot \underline{\tilde{T}}^a + \underline{\omega}^a{}_b \cdot \underline{\tilde{T}}^b := \underline{v}^b \cdot \underline{\tilde{R}}^a{}_b \quad -(27)$$

for space like indices:

$$\mu, \nu, \rho = i, j, k = 1, 2, 3 \quad -(28)$$

i.e.

$$D_1 T^a{}_{23} + D_2 T^a{}_{31} + D_3 T^a{}_{12} := R^a{}_{123} + R^a{}_{231} + R^a{}_{312} \quad -(29)$$

and:

$$D_1 \tilde{T}^a{}_{23} + D_2 \tilde{T}^a{}_{31} + D_3 \tilde{T}^a{}_{12} := \tilde{R}^a{}_{123} + \tilde{R}^a{}_{231} + \tilde{R}^a{}_{312} \quad -(30)$$

When the time like or 0 index of eqs. (24) and (25) is considered, the cyclic sum (29) becomes:

$$D_0 T^a{}_{23} + D_2 T^a{}_{30} + D_3 T^a{}_{02} := R^a{}_{023} + R^a{}_{230} + R^a{}_{302}$$
$$D_0 T^a{}_{31} + D_1 T^a{}_{03} + D_3 T^a{}_{10} := R^a{}_{031} + R^a{}_{103} + R^a{}_{310} \quad -(31)$$
$$D_0 T^a{}_{12} + D_1 T^a{}_{20} + D_2 T^a{}_{01} := R^a{}_{012} + R^a{}_{120} + R^a{}_{201}$$

and its Hodge dual equivalent.

For eqs. (31) it is again true that if torsion is zero, curvature is zero. This will be discussed in the next note.

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