1) 250(5): Proof of the ESOR Term with the Pauli Matrices

The commutator relations of spin are proven with the Pauli matrices:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(1)$$

$$S_x = \frac{1}{2}\hbar\sigma_x, \quad S_y = \frac{\hbar}{2}\sigma_y, \quad S_z = \frac{\hbar}{2}\sigma_z \quad -(2)$$

It follows ~~that~~:

$$[S_x, S_y]\phi = i\hbar S_z \phi \quad -(3)$$

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The ESOR term follows immediately from the use of the Pauli matrices. Consider:

$$\underline{\sigma} \cdot \underline{L} = \begin{bmatrix} L_z & 0 \\ 0 & -L_z \end{bmatrix} \quad -(4)$$

$$\underline{\sigma} \cdot \underline{B} = \begin{bmatrix} B_z & 0 \\ 0 & -B_z \end{bmatrix} \quad -(5)$$

and

$$\phi = \phi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad -(6)$$

then

$$\underline{\sigma} \cdot \underline{L}\, \underline{\sigma} \cdot \underline{B}\, \phi = \begin{bmatrix} L_z & 0 \\ 0 & -L_z \end{bmatrix} \begin{bmatrix} B_z & 0 \\ 0 & -B_z \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \phi$$

$$= \begin{bmatrix} L_z B_z & 0 \\ 0 & L_z B_z \end{bmatrix} \phi \quad -(7)$$

$$= L_z B_z \phi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now consider:

2)

$$\boldsymbol{\sigma}\cdot\mathbf{L}\,\phi\,\boldsymbol{\sigma}\cdot\mathbf{B}$$

$$= \begin{bmatrix} L_Z & 0 \\ 0 & -L_Z \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\phi\begin{bmatrix} B_Z & 0 \\ 0 & -B_Z \end{bmatrix}$$

$$= \begin{bmatrix} L_Z B_Z & 0 \\ 0 & L_Z B_Z \end{bmatrix}\phi \quad -(8) \quad = L_Z B_Z\,\phi\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So:

$$\boxed{\begin{aligned} \boldsymbol{\sigma}\cdot\mathbf{L}\,\boldsymbol{\sigma}\cdot\mathbf{B}\,\phi &= \boldsymbol{\sigma}\cdot\mathbf{L}\,\phi\,\boldsymbol{\sigma}\cdot\mathbf{B} \\ &= \frac{2}{\hbar}\left(\mathbf{S}\cdot\mathbf{L}\right)\phi\,\boldsymbol{\sigma}\cdot\mathbf{B} \end{aligned}} \quad -(9)$$

This equation is quantized to give the ESOR hamiltonian:

$$\boxed{\hat{H}_{ESOR}\,\phi = -\frac{e}{m\hbar}\left(\hat{\mathbf{S}}\cdot\hat{\mathbf{L}}\right)\phi\,\boldsymbol{\sigma}\cdot\mathbf{B}} \quad -(10)$$

This is a direct and simple derivation without use of the Leibnitz theorem. The use of the SU(2) basis introduces spin in an entirely new way.

The Zeeman effect hamiltonian is:

$$\hat{H}_{Zeeman} = -\frac{e}{2m}\,\hat{\mathbf{L}}\cdot\mathbf{B}\,\phi \quad -(11)$$

$$\neq -\frac{e}{2m}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{L}}\,\boldsymbol{\sigma}\cdot\mathbf{B}\,\phi$$