250(4): Comparison of Electron Spin Orbit Resonance Frequencies with those of the Zeeman Effect and Anomalous Zeeman Effect.

From eq. (15) of note 250(3) the complete hamiltonian is:

$$\hat{H}\psi = -\frac{e}{2m}\left(g_L \underline{\hat{J}}\cdot\underline{B}\psi + \frac{2}{\hbar}\left(\underline{\hat{S}}\cdot\underline{\hat{L}}\psi\right)\underline{\sigma}\cdot\underline{B}\right) \quad -(1)$$

Anomalous Zeeman Effect and Zeeman Effect

Here

$$\hat{J}_Z\psi = m_J\hbar\psi \quad -(2)$$

and the Landé factor is:

$$g_L = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad -(3)$$

The resonance frequencies in the anomalous Zeeman effect are:

$$\hbar\omega = \frac{e}{2m} g_L m_J \hbar \quad -(4)$$

so

$$\boxed{\begin{aligned} \omega_{AZ} &= \frac{e}{2m} g_L m_J \\ m_J &= -J, \ldots, J \\ J &= L+S, \ldots, |L-S| \end{aligned}} \quad -(5)$$

If the electron is in a state where:

1)
$$S = 0 \quad - (6).$$

then:
$$g_L = 1 \quad - (7)$$

and the resonance frequencies of the normal Zeeman effect are:
$$\boxed{\omega_Z = \frac{e}{2m} m_L} \quad - (8)$$

where
$$m_L = -L, \ldots, L \quad - (9)$$

<u>Electron Spin Orbit Resonance</u>

the relevant Hamiltonian is:
$$\hat{H}_{ESOR} \psi = -\frac{e}{m\hbar} \left(\underline{\hat{S}} \cdot \underline{\hat{L}} \psi \right) \underline{\sigma} \cdot \underline{B} \quad - (10).$$

and resonance occurs between the states of
$$\sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad - (11)$$

so
$$\omega_{ESOR} \psi = \frac{2e}{m\hbar^2} \left(\underline{\hat{S}} \cdot \underline{\hat{L}} \psi \right) \quad - (12)$$

In the coupled representation in a strong external magnetic field:
$$\underline{\hat{S}} \cdot \underline{\hat{L}} \psi = \hat{S}_Z \hat{L}_Z \psi \quad - (13)$$
$$= \hbar^2 m_S m_L \psi$$

3)

So:

$$\boxed{\omega_{ESOR} = 2 m_s m_L \frac{eB}{m}} \quad -(14)$$

In the uncoupled representation:

$$\hat{\underline{S}} \cdot \hat{\underline{L}} \psi = \frac{1}{2} \hbar^2 \left(J(J+1) - L(L+1) - S(S+1) \right) \quad -(15)$$

So

$$\boxed{\begin{array}{l} \omega_{ESOR} = \left(\left(J(J+1) - L(L+1) - S(S+1) \right) \right) \dfrac{eB}{m} \quad -(16) \\ J = L+S, \ldots, |L-S| \quad -(17) \end{array}}$$

Summary

The ESOR term is obtained from the Pauli algebra:

$$\underline{L} \cdot \underline{B} = \underline{\sigma} \cdot \underline{L} \, \underline{\sigma} \cdot \underline{B} \quad -(18)$$

Eq. (18) follows from:

$$\underline{\sigma} \cdot \underline{L} \, \underline{\sigma} \cdot \underline{B} = \underline{L} \cdot \underline{B} + i \underline{\sigma} \cdot \underline{L} \times \underline{B} \quad -(19)$$

If $\underline{L}$ and $\underline{B}$ are parallel:

$$\underline{L} \cdot \underline{B} = L_Z B_Z \quad -(20)$$

then:

$$\underline{L} \times \underline{B} = \underline{0} \quad -(21)$$

and eq. (18) follows, QED.

4) In quantum mechanics $\underline{\sigma}$ becomes a vector operator defined by:

$$\underline{S} = \frac{1}{2}\hbar\,\underline{\sigma} \quad -(22)$$

It follows that the use of the SU(2) basis for $\underline{L}\cdot\underline{B}$ introduces the ESOR term in quantum mechanics:

$$\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\hat{\underline{\sigma}}\cdot\underline{B}\,\phi = \left(\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\phi\right)\hat{\underline{\sigma}}\cdot\underline{B} + \left(\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\hat{\underline{\sigma}}\cdot\underline{B}\right)\phi \quad -(23)$$

Using eq. (18):

$$\left(\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\hat{\underline{\sigma}}\cdot\underline{B}\right)\phi = \left(\hat{\underline{L}}\cdot\underline{B}\right)\phi \quad -(24)$$

So:

$$\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\hat{\underline{\sigma}}\cdot\underline{B}\,\phi = \left(\hat{\underline{L}}\cdot\underline{B}\right)\phi + \left(\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\phi\right)\hat{\underline{\sigma}}\cdot\underline{B}$$

$$= \left(\hat{\underline{L}}\cdot\underline{B}\right)\phi + \left(\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\phi\right)\underline{\sigma}\cdot\underline{B} \quad -(25)$$

$$= \underbrace{\left(\hat{\underline{L}}\cdot\underline{B}\right)\phi}_{\text{Zeeman effect}} + \underbrace{\frac{2}{\hbar}\left(\hat{\underline{S}}\cdot\hat{\underline{L}}\,\phi\right)\underline{\sigma}\cdot\underline{B}}_{\text{electron spin orbit effect}}$$