250(3): Development of the Anomalous Zeeman Effect into Electron Spin Orbit Resonance

Consider the Hamiltonian of the anomalous Zeeman effect:

$$\hat{H}\psi = -\frac{e}{2m}\left(\underline{L}\cdot\underline{B}\,\psi + \hbar\,\underline{\sigma}\cdot\underline{B}\,\psi\right)$$

$$= -\frac{e}{2m}\,g_L\,\underline{J}\cdot\underline{B} \quad -(1)$$

where g_L is the Landé factor defined in the previous note. Here

$$\underline{S} = \frac{1}{2}\hbar\,\underline{\sigma} \quad -(2)$$

If $\underline{L}$ and $\underline{B}$ are real valued:

$$\underline{\sigma}\cdot\underline{L}\;\underline{\sigma}\cdot\underline{B} = \underline{L}\cdot\underline{B} \quad -(3)$$

otherwise:

$$\underline{\sigma}\cdot\underline{L}\;\underline{\sigma}\cdot\underline{B} = \underline{L}\cdot\underline{B} + i\,\underline{\sigma}\cdot\underline{L}\times\underline{B} \quad -(4)$$

Now consider:

$$\underline{\sigma}\cdot\underline{L}\;\underline{\sigma}\cdot\underline{B}\;\psi = \frac{4}{\hbar^2}\,\underline{S}\cdot\underline{L}\;\underline{S}\cdot\underline{B}\,\psi \quad -(5)$$

from eq. (2). Note carefully that $\underline{S}$ is a vector operator.

Using the Leibnitz theorem:

2)

$$\underline{S}\cdot\underline{L}\,\underline{S}\cdot\underline{B}\,\phi = \left(\underline{S}\cdot\underline{L}\,\underline{S}\cdot\underline{B}\right)\phi + \left(\underline{S}\cdot\underline{L}\,\phi\right)\underline{S}\cdot\underline{B} \quad -(6)$$

Therefore:

$$\underline{\sigma}\cdot\underline{L}\,\underline{\sigma}\cdot\underline{B}\,\phi = \underline{L}\cdot\underline{B}\,\phi + \frac{4}{\hbar^2}\left(\underline{S}\cdot\underline{L}\,\phi\right)\underline{S}\cdot\underline{B}$$

$$= \underline{L}\cdot\underline{B}\,\phi + \frac{2}{\hbar}\left(\underline{S}\cdot\underline{L}\,\phi\right)\underline{\sigma}\cdot\underline{B} \quad -(7)$$

Now use:

$$\underline{S}\cdot\underline{L}\,\phi = \frac{\hbar^2}{2}\left(J(J+1)-L(L+1)-S(S+1)\right) \quad -(8)$$

where

$$J = L+S, \ldots, |L-S| \quad -(9)$$

Therefore:

$$\underline{\sigma}\cdot\underline{L}\,\underline{\sigma}\cdot\underline{B}\,\phi = \underline{L}\cdot\underline{B}\,\phi + \left(J(J+1)-L(L+1)-S(S+1)\right)\hbar\,\underline{\sigma}\cdot\underline{B} \quad -(10)$$

and:

$$\underline{L}\cdot\underline{B}\,\phi = \underline{\sigma}\cdot\underline{L}\,\underline{\sigma}\cdot\underline{B}\,\phi - \left(J(J+1)-L(L+1)-S(S+1)\right)\hbar\,\underline{\sigma}\cdot\underline{B} \quad -(11)$$

From eqs. (10) or (11) there is a new type

3) of resonance which occurs at:

$$\boxed{\omega_{ESOR} = \left(J(J+1) - L(L+1) - S(S+1) \right) \frac{eB}{m}} \quad -(12)$$

where

$$J = L + S, \ldots, |L - S| \quad -(13)$$

This is named electron spin orbit resonance.

It occurs when the hamiltonian of the anomalous Zeeman effect is written as:

$$\hat{H}\psi = -\frac{e}{2m}\left(\underline{\sigma} \cdot \underline{L}\, \underline{\sigma} \cdot \underline{B}\, \psi + \hbar\, \underline{\sigma} \cdot \underline{B}\, \psi \right) \quad -(14)$$

Eq. (14) gives:

$$\hat{H}\psi = -\frac{e}{2m} g_L\, \underline{J} \cdot \underline{B} - \frac{2}{\hbar}\left(\underline{S} \cdot \underline{L}\, \psi \right) \underline{\sigma} \cdot \underline{B} \quad -(15)$$

The reason is that:

$$\underline{L} \cdot \underline{B}\, \psi \neq \underline{\sigma} \cdot \underline{L}\, \underline{\sigma} \cdot \underline{B}\, \psi \quad -(16)$$

because in quantum mechanics:

$$\underline{S} = \frac{1}{2} \hbar\, \underline{\sigma} \quad -(17)$$

is an operator.